Strong interaction

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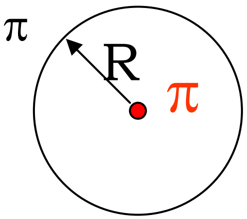
1. Strong isospin

1.1 Strong interaction characteristics :

Geometrical size of the π

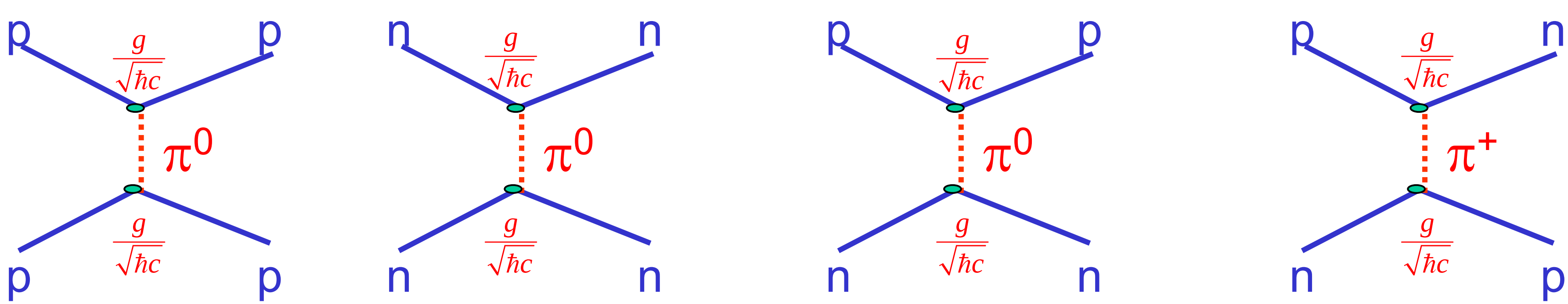


- Order of magnitude of the strong cross section in the nucleon-nucleon diffusion :

$$\sigma_{forte} \sim 30\text{mb} \sim \pi \left(\frac{\hbar c}{m_\pi c^2} \right)^2$$

- « Strong » interaction :
  typical binding energies : 2 to 8 MeV, coupling constant : $g^2/\hbar c \sim 0.1$

- Historical approach (Yukawa) : $\pi^\pm$ and π^0 are the strong interaction vectors (range ~ 1 fm $\Rightarrow$ existence of a field quantum with a mass ~200 MeV)



Experimentally : strong interactions do not depend on the electric charge (same intensity for np, nn and pp reactions) $\Rightarrow$ π exchange of same mass

1.2 The rotation group and the SU(2) group :

Reminder:

- Rotation group : 3 generators J_i
- The generators for the rotations constitute an algebra:

$$\left[J_i, J_k\right] = i\varepsilon_{ikl} J_l$$

$\varepsilon_{ikl} = +1(-1)$ if *ikl* is a cyclic permutation (anti-cyclic) of 1,2,3

$= 0$ In the other cases

- These relations define the properties of the group.
- Properties:

- 2 J_i operators do not commute

- one can define an operator (so-called Casimir operator) : non-linear combination of the generators which commute with all generators :

$$J^2 = J_1^2 + J_2^2 + J_3^2 \quad ; \quad \left[J^2, J_i\right] = 0$$

From these 2 properties one deduces that only 2 operators are independent (2 useful quantum numbers). One usually chooses J^2 and J_z (or J_3)

Representation of the rotations group:

- In a given representation of the rotations group (tagged by the J^2 eigenvalue J), one can build the eigen-vectors $|j\ m>$:

$$J^2 |jm\rangle = J(J+1)|jm\rangle \qquad J = 0, 1/2, 1, \ldots$$
$$J_z |jm\rangle = m|jm\rangle \qquad m = -J, -J+1, \ldots, J$$

- The representation which corresponds to a given value of *J* is of dimension 2*J*+1 (*m* can take all values between –*J* and +*J*)
- One can build all eigen-vectors of this representation from one (here |j,m>) thanks to the operators $J_\pm = J_x \pm iJ_y$ using the relationships :

standard basis

$$J_+|j,m\rangle = \hbar\sqrt{J(J+1)-m(m+1)}\,|j,m+1\rangle$$
$$J_-|j,m\rangle = \hbar\sqrt{J(J+1)-m(m-1)}\,|j,m-1\rangle$$

- One can describe the rotation operators by unitary matrices of size $(2J+1)\times(2J+1)$ in the standard basis.

Representation of an operator in a basis = a matrix

=> one gets the rotation matrices (unitary matrices, with determinant=+1).

Example : dimension 2 representation, spin ½ particles

- J = 1/2 , J_3=-½, +½: dimension 2 representation of the rotations group
- We call it SU(2)
- By convention the eigenstates of S_z (corresponding to J_z) are used as a basis:

$$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{S}_+ = \hbar \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \hat{S}_- = \hbar \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$= S_x+iS_y$ $\qquad$ $= S_x-iS_y$

Pauli matrices

- The kinetic moments operators are :

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

thus $\hat{S}_i = \frac{\hbar}{2}\sigma_i$

- There are several representations for the rotations group

Dimension for the representation	J, eigenvalue of J^2
1	0
2	1/2 → SU(2) : corresponds to σ_i matrices
3	1 → SO(3): rotations in ordinary space
4	3/2

- SU(2) is the fundamental representation of the rotations group : from SU(2) one can build all the other presentations (we usually use the name SU(2) to talk about the rotations group)
- Settings of the representations :

One usually writes it using dimensions

$\boxed{\begin{array}{l} 1/2\otimes 1/2 = 0\oplus 1 \\ 1/2\otimes 1/2\otimes 1/2 = (0 \oplus 1)\otimes 1/2 = 1/2 \oplus 3/2 \end{array}} \Longleftrightarrow \boxed{\begin{array}{l} 2\otimes 2 = 1\oplus 3 \\ 2\otimes 2\otimes 2 = 2\oplus 4 \end{array}}$

- One can build the multiplets from the fundamental mutiplet ½ (dim. 2)

1.3 The n-p system, strong isospin definition :

We are going to build a model for the n-p system (from the strong interaction point of view) starting from the 3 following experimental facts :

a) The electric charge Q is conserved in n-p interactions:

$pn \rightarrow pn$ $\cancel{pn \rightarrow nn}$

$Q_i = Q_f$ never observed

b) The n-p interactions are charge independent (~1936) :

from the strong interaction point of view : $pn \equiv pp \equiv nn$

(if one neglects the electromagnetic interaction)

c) The proton and neutron masses are very similar :

$M_n = 939.56$ MeV

$M_p = 938.27$ MeV

The strong interaction does not distinguish between proton and neutron : we are going to assume that they are identical (nucleon N)

Model building for the n-p system

- n and p are two levels of a quantum system.
- The eigenvectors are :

$$|n\rangle \quad |p\rangle \qquad \begin{pmatrix}0\\1\end{pmatrix} \quad \begin{pmatrix}1\\0\end{pmatrix}$$

- The interaction hamiltonian: $H_{tot}=H_{strong}+\cancel{H_{elm}}$ neglected

Taking into account the experimental facts :

a) <u>The electric charge is conserved:</u>

We first look for the hermitian operator Q such that : Q|n>=0|n> et Q|p>=1|p>

The representation of Q in {|*p*>,|*n*>} is : $Q=\begin{pmatrix}1&0\\0&0\end{pmatrix}$

Using $\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$, one can write: $\hat{Q}=\frac{1}{2}\sigma_3+\frac{\mathbf{1}}{2}$

One defines the operator I_3 such that: $\boxed{\hat{Q}=\hat{I}_3+\frac{\mathbf{1}}{2} \quad ; \quad \hat{I}_3=\frac{1}{2}\sigma_3}$

In order to have charge conservation one should impose : $[H_{Strong},Q]=0$

$$\Leftrightarrow [H_{Strong},\sigma_3]= [H_{Strong},I_3]= 0$$

b) Charge independence implication :

By analogy with the operators + and – for the spin (S_+ and S_-) :

$$\begin{cases} I_+|n\rangle = |p\rangle \\ I_-|p\rangle = |n\rangle \end{cases} \quad \text{and} \quad \begin{cases} I_+|p\rangle = 0 \\ I_-|n\rangle = 0 \end{cases}$$

Charge independence $\Rightarrow$ H does not change under the action of the operators which transform |*p*> in |*n*> and |*n*> in |*p*>, which are the operators $I_\pm$

$$\left[H_{Strong}, I_\pm\right] = 0 \qquad \Rightarrow \left[H_{Strong}, I_1\right] = \left[H_{Strong}, I_2\right] = \left[H_{Strong}, I^2\right] = 0$$

One can write:

$$I_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad ; \quad I_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad \text{in the space } \{|p\rangle, |n\rangle\}$$

Using :
$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

One gets :
$$I_+ = \frac{1}{2}(\sigma_1 + i\sigma_2) \quad I_- = \frac{1}{2}(\sigma_1 - i\sigma_2) \Rightarrow \left[H_{Strong}, \sigma_1\right] = \left[H_{Strong}, \sigma_2\right] = 0$$

With a), one finds : $\left[H_{Strong}, \sigma_i\right] = 0 \qquad i = 1,2,3$

The strong interaction is invariant under the symmetry |n> ↔ |p>. This symmetry is SU(2) (generators: σ_i). We call it strong isospin symmetry.

This symmetry we just have defined also predicts the third experimental fact c) :

$$I_+|n\rangle = |p\rangle \quad \Rightarrow HI_+|n\rangle = H|p\rangle \quad \Rightarrow I_+H|n\rangle = H|p\rangle$$

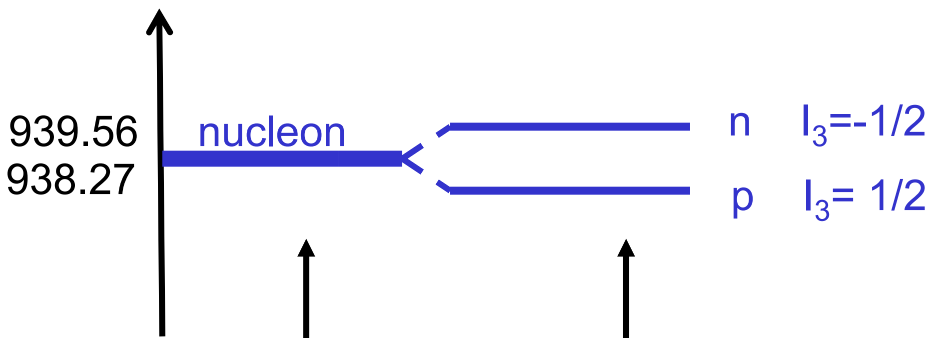
$$\Rightarrow m_n I_+|n\rangle = m_p|p\rangle \quad \Rightarrow m_n|p\rangle = m_p|p\rangle$$

$$\Rightarrow m_n = m_p$$

One uses : H|p>=m|p>

The H eigenvalue is the mass of |p>

Schematic view of the symmetry :



H_{int}= H_{strong} + H_{elm}

invariant under SU(2)-isospin

$[H_{strong},I_\pm]= [H_{strong},I_3]=0$

SU(2)-isospin is broken (small effect)

The electromagnetic interaction:

- depends on the charge
  $n \neq p \Rightarrow [H_{elm},I_\pm] \neq 0$

- conserve the charge
  $[H_{elm},I_3] = 0$

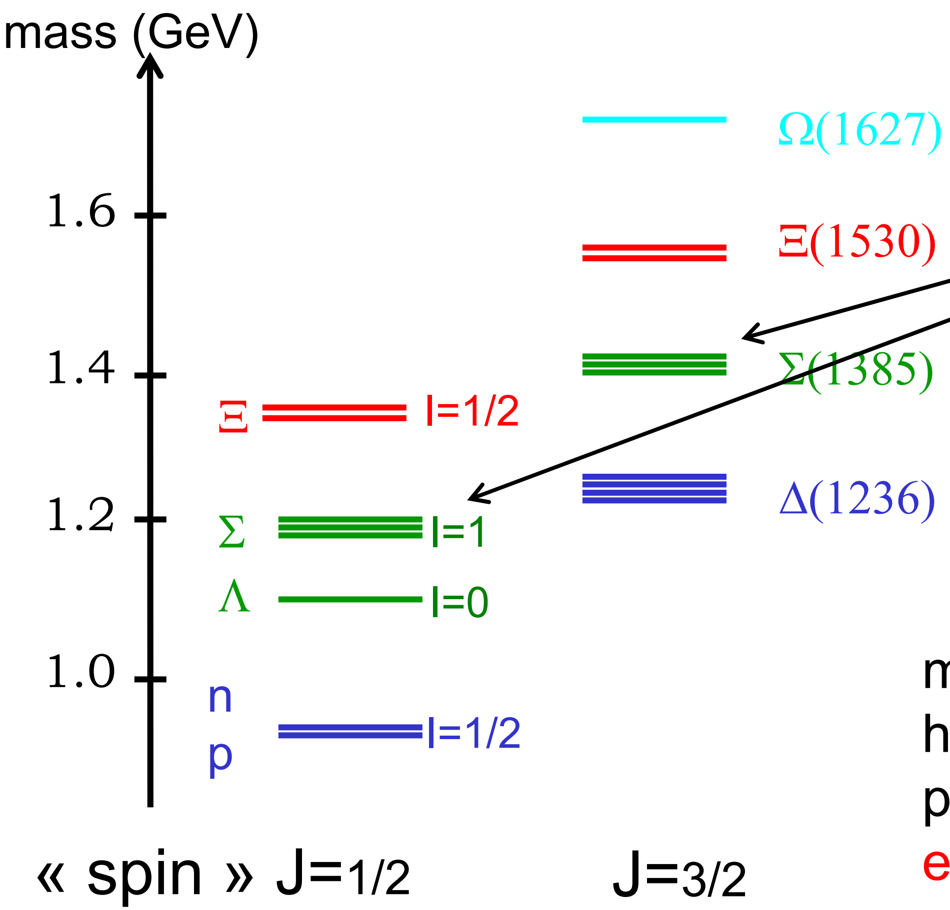
$$\hat{Q} = \hat{I}_3 + \frac{1}{2}$$

1.4 Isospin multiplets :

• Experimentally : isospin is conserved in the π-Nucleons interactions

⇒ Extension of the isospin formalism, first to the pion: (π^+ π^- π^0) triplet , isospin 1, then to all particles undergoing the strong interaction

⇒mesons and baryons are grouped into isospin multiplets :



Different values of J =>different masses (due to the S▪S ; S▪L terms…)

multiplets definition: «groups» of particles having the same quantum numbers (spin, parity) roughly the same mass but different electric charges

Summary: strong isospin

- From the experimental observation (|n> $\leftrightarrow$ |p>), one has introduced a 2 levels system similar to the spin (same algebra): : $[H_{strong},\sigma_i] = 0$.
  SU(2) is a symmetry group of H_{strong} (invariance $\leftrightarrow$ symmetry).
  This symmetry is called isospin symmetry.

- {|*p*>,|*n*>} generates the isospin space in dimension 2 (I=1/2).
  H_{strong} is invariant by rotation in this space.

- Isospin conservation works in a way similar to the kinetic momentum **J** conservation (same algebra: $[J_i,J_j] = i\varepsilon_{ijk}J_k$).
- Mesons and baryons can be grouped into isospin multiplets
  (I=0, I=1/2, I=1, …).

- *Note : we will see later there there is another type of isospin : the weak isospin. Usually when one speaks about isospin it means strong isospin*

2. Baryonic number, strangeness, flavour-SU(3)

2.1 The baryonic number :

- Experimentally one does NOT see :

$$pp \rightarrow \text{mesons or leptons}$$

(for any interaction)

$\Rightarrow$ One makes the hypothesis of the existence of a new quantum number (an additive one): the baryonic number

$$\boxed{\begin{array}{ll} \hat{B}|p\rangle = |p\rangle & \hat{B}|\bar{p}\rangle = -|\bar{p}\rangle \\ \hat{B}|n\rangle = |n\rangle & \hat{B}|\bar{n}\rangle = -|\bar{n}\rangle \\ \hat{B}|\pi\rangle = 0 & \\ \hat{B}|\text{lepton}\rangle = 0 & \end{array}}$$

This − sign comes the observation that $p\bar{p} \rightarrow mesons$...

Baryonic number and isospin

We had: $\boxed{\hat{Q} = \hat{I}_3 + \frac{1}{2} \quad \text{with} \quad p,n}$

Writing : $\boxed{\hat{Q} = \hat{I}_3 + \frac{\hat{B}}{2}}$

One gets :

	n	p	π^+	π^0	π^-
Isospin: I	1/2		1		
I_3	-1/2	1/2	1	0	-1
Baryonic number : B	1	1	0	0	0
Charge: Q	0	1	1	0	-1

Thus the expression $Q=I_3+B/2$ works also now for the π.

2.2 Strangeness :

- ~1947 : new particles discovery in cosmic rays

  K (~500 MeV) Λ (~1100 MeV)

- Why strange ?

  The K and Λ production cross section is of the same order of magnitude as the one of the other known hadrons at that time (eg π).

  Their lifetime is surprisingly large wrt the time scale of strong interaction (even with quite large masses)

  Different lifetimes $\Rightarrow$ different interactions in the decay

  The proposed explanation is that they are produced by strong interaction (as the π) but that they decay via weak interaction.

- Why don't they decay via strong interaction or elm interaction ?

  Something should prevent these interactions in the decay

•Pais's intuition (1952):

A new quantum number exists

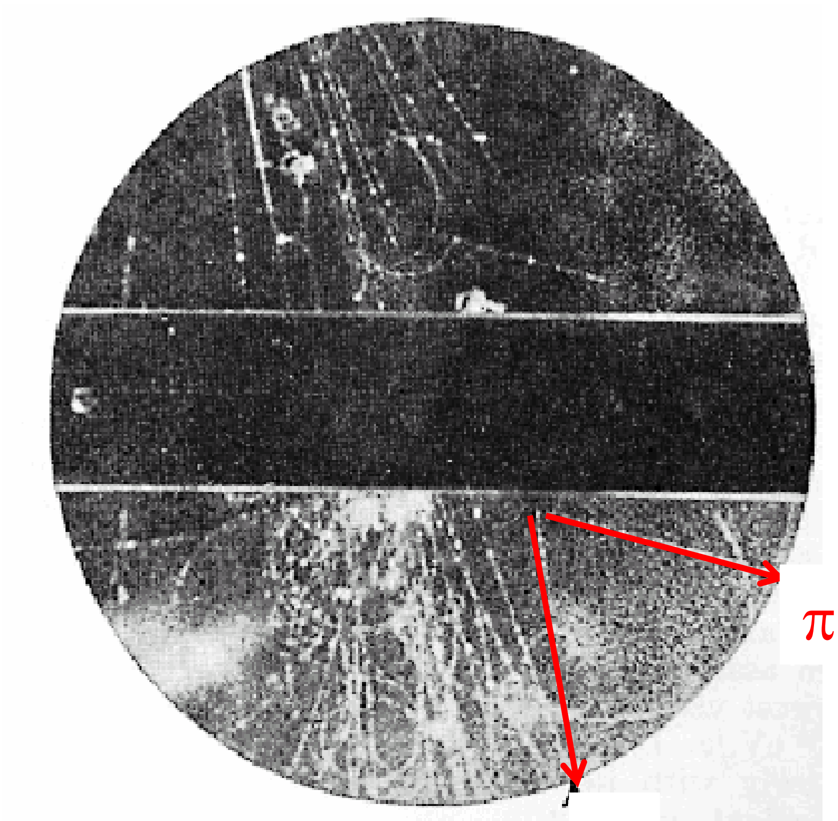
- Conserved by strong interaction
- Non conserved by weak interaction

$\boxed{\text{strangeness}}$

Experimental signs of strange particles : the K

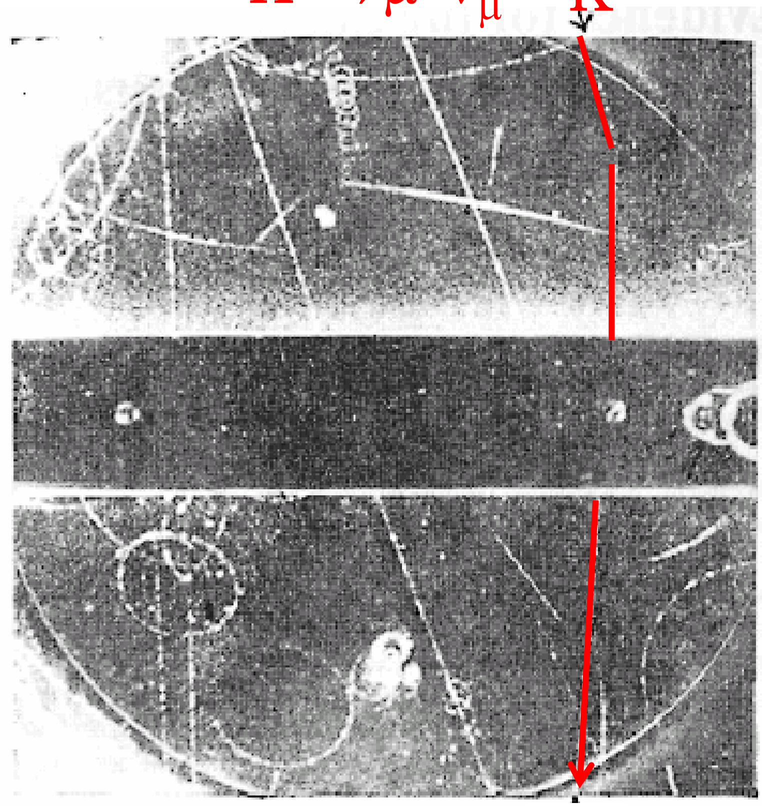
Cloud chamber ~1947

$K^0 \rightarrow \pi^+\pi^-$



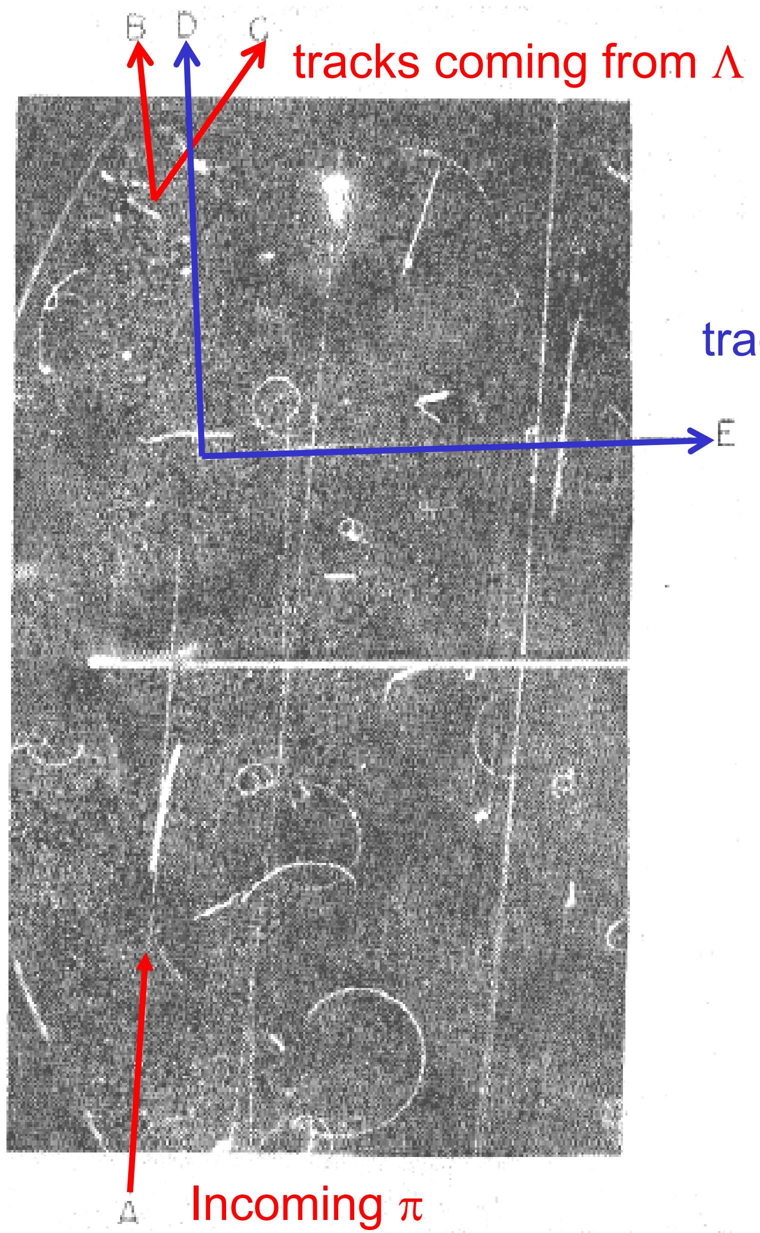
V-particle

$K^+ \rightarrow \mu^+\nu_\mu$



«Kink» in the detector

Experimental signs of strange particles : the Λ



1955 Walker *et al.*
(Berkeley)

$$\pi^- \, p \rightarrow K^0 \Lambda^0$$

Pair production

FIG. 1. $\Lambda^0-\theta^0$ production in a $\pi^- - P$ collision. Track *A* is the incoming π^- meson which disappears in flight. Direct measurements on this track give a momentum between 1.05 and 1.3 Bev/*c*. The adjacent π^- meson which crosses the chamber has a momentum of 1.14±0.10 Bev/*c*. Tracks *B* and *C* are the decay products of a Λ^0. Track *C* is short but momentum measurements give a momentum of less than 100 Mev/*c* and a negative sign. Tracks *D* and *E* are the π^- and π^+ mesons from the decay of the θ^0. Measurements on the π^+ meson give 153±8 Mev/*c* for the momentum.

Production/ decay of Λ

- Production: $\pi^- p \to K^0 \Lambda^0$. Cross section ~ mb $\Rightarrow$ strong interaction (pair production of the strange particles)
- decay : $\Lambda^0 \to p\pi^-$ and $\Lambda^0 \to n\pi^0$: $\tau(\Lambda^0)=2.6\ 10^{-10}$ s

  (typical values τ_{strong}~ 10^{-23} s τ(elm) ~ 10^{-16} s) $\Rightarrow$ weak interaction

Initial state isospin = Λ isospin ; Pion isospin : 1 ; Proton isospin : ½ $\Rightarrow$ final state isospin : ½ or 3/2
Hypothesis : it is not a strong interaction process $\Rightarrow$ isospin is violated $\Rightarrow$ slow down the decay which occurs via elm interaction
Let's try $I(\Lambda)=5/2 \Rightarrow I_3(\Lambda)=5/2, 3/2, ½, -1/2, -3/2, -5/2$

I^2 is not conserved : isospin partially broken but I_3 is conserved (the elm interaction conserves the electric charge !)

The final state can have $I_3(\pi p)$= 3/2, ½, -1/2, -3/2
$\Rightarrow$ decay via elm interaction is possible … but lifetime !

Assume a large isospin value to explain the decay of strange particles does not work …

$I(\Lambda) = 0$

particle	strangeness: S
$p, n, \pi^\pm, \pi^0$	0
$\Lambda, \Sigma^\pm, \Sigma^0$	-1
$\Xi^\pm, \Xi^0$	-2
K^0, K^+	+1
$\overline{K}^0, K^-$	-1

$\Rightarrow$Additive quantum number : strangeness as we have just done for the baryonic number

strangeness :
- conserved by strong and electromagnetic interactions
- not conserved by weak interaction

Definition of the strange isospin multiplets d'isospin

Two quantum numbers : I_3 and S $\Rightarrow$ widening of the isospin symmetry

Caution ! S means strangeness not spin!

$$\boxed{SU(2) \to \{I_3, S\} \equiv SU(3)}$$

If there is a SU(3) symmetry, the hadrons with similar properties should be grouped into SU(3) multiplets

By convention one speaks in terms of I_3 and hypercharge multiplets

Definition :

The charge of strange hadrons is written as :

$$\boxed{\hat{Q} = \hat{I}_3 + \frac{\hat{B}}{2} + \frac{\hat{S}}{2}}$$

One defines the operator hypercharge (Y) as :

$$\boxed{\hat{Y} \equiv \hat{B} + \hat{S} \quad ; \quad \hat{Q} = \hat{I}_3 + \frac{\hat{Y}}{2}}$$

$\hat{Q}, \hat{I}_3, \hat{B}, \hat{S}$	conserved by strong and electromagnetic interactions
$\hat{Q}, \hat{B}$	conserved by weak interaction
$\hat{I}_3, \hat{S}$	Violated by weak interaction

2.3 The SU(3) group and the flavour-SU(3) symmetry :

SU(3) group :

- Set of unitary 3x3 matrices with determinant 1
- Group generators : $3^2-1=8$ independent matrices with a null trace
- Only 2 of those matrices are diagonal
  = maximum number of commutating generators (the rank of SU(3) is 2).

There are 8 generators

λ_i the Gell-Mann matrices :

$$\lambda_1=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}\quad \lambda_2=\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}\quad \lambda_3=\begin{pmatrix}\boxed{\begin{matrix}1&0\\0&-1\end{matrix}}&\begin{matrix}0\\0\end{matrix}\\\begin{matrix}0&0\end{matrix}&0\end{pmatrix}\ \sigma_3$$

$$\lambda_4=\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}\quad \lambda_5=\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}\quad \lambda_6=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

$$\lambda_7=\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}\quad \lambda_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

Useful for the quarks model (see later)

σ_3

$$I_3=\frac{1}{2}\begin{pmatrix}\boxed{\begin{matrix}1&0\\0&-1\end{matrix}}&\begin{matrix}0\\0\end{matrix}\\\begin{matrix}0&0\end{matrix}&0\end{pmatrix}$$

$$\begin{cases} I_3 = Q-Y/2\ (S\to S\ ;\ Q\to Q) \\ I_\pm\ (S\to S)\ (Q\to Q\pm1) \end{cases}$$

$$\begin{cases} Y = B+S\ (S\to S\ ;\ Q\to Q) \\ U_\pm\ (Q\to Q\ ;\ S\to S\pm1) \\ V_\pm\ (Q\to Q\pm1\ ;\ S\to S\pm1) \end{cases}$$

Allow to move within a multiplet (see the graphical representation latter)

Representations and multiplets of SU(3)

- There are 2 fundamental representations of SU(3) : the triplets **3** and $\bar{\mathbf{3}}$

  For SU(3): $\bar{3}$ is not the same representation as 3 (whereas for SU(2): $2=\bar{2}$)

- combinations:

  $3\otimes3\otimes3 = 1\oplus8\oplus8\oplus10$
  $3\otimes3\otimes\bar{3} = 3\oplus3\oplus6\oplus15$
  $3\otimes\bar{3} = 1\oplus8$

  Writings in terms of dimension
  $\frac{1}{2}\otimes\frac{1}{2} \leftrightarrow 2\otimes2$

  One should look at it as the sum of 2 spins
  (for example $\frac{1}{2}\otimes\frac{1}{2} \rightarrow$ one singlet and one triplet : $2\otimes2=1\oplus3$)

- experimentally, for flavour-SU(3):

  The baryons are grouped into multiplets of multiplicity

  1 8 10 (obtained from $3\otimes3\otimes3$)

  The mesons are grouped into multiplets of multiplicity

  1 8 (obtained from $3\otimes\bar{3}$)

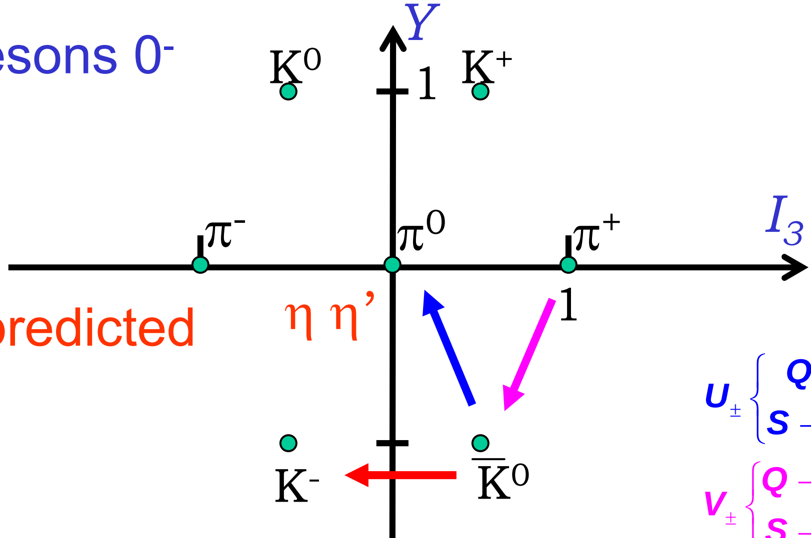
  and nothing else !

It cannot be random !!
(see later)

Few flavour-SU(3) multiplets

mesons 0^-

η predicted

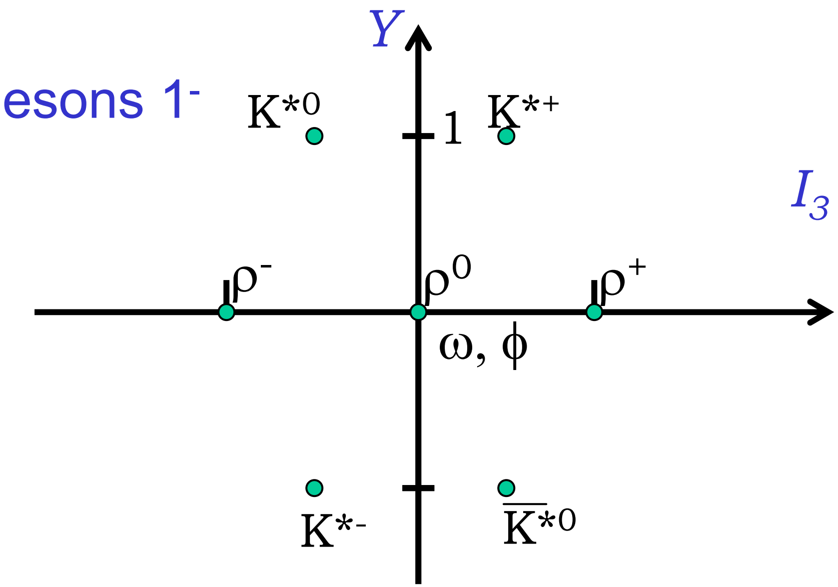


$$U_\pm \begin{cases} Q \to Q \\ S \to S \pm 1 \end{cases}$$

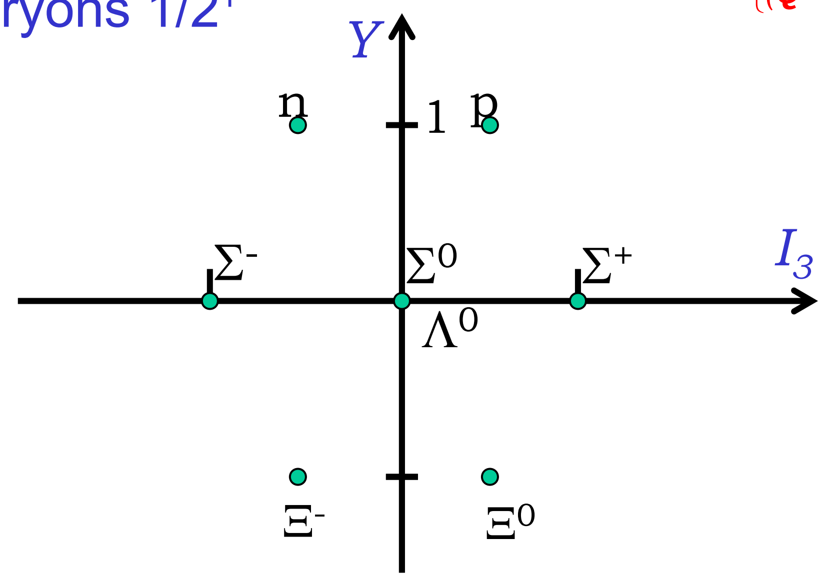
$$V_\pm \begin{cases} Q \to Q \pm 1 \\ S \to S \pm 1 \end{cases}$$

$$I_\pm \begin{cases} (S \to S) \\ (Q \to Q \pm 1) \end{cases}$$

mesons 1^-

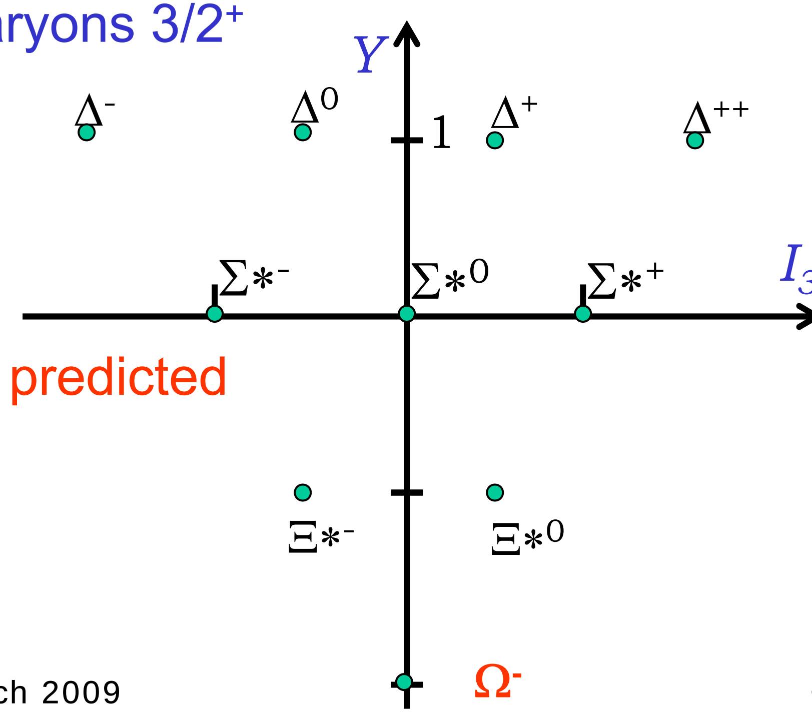


baryons $1/2^+$

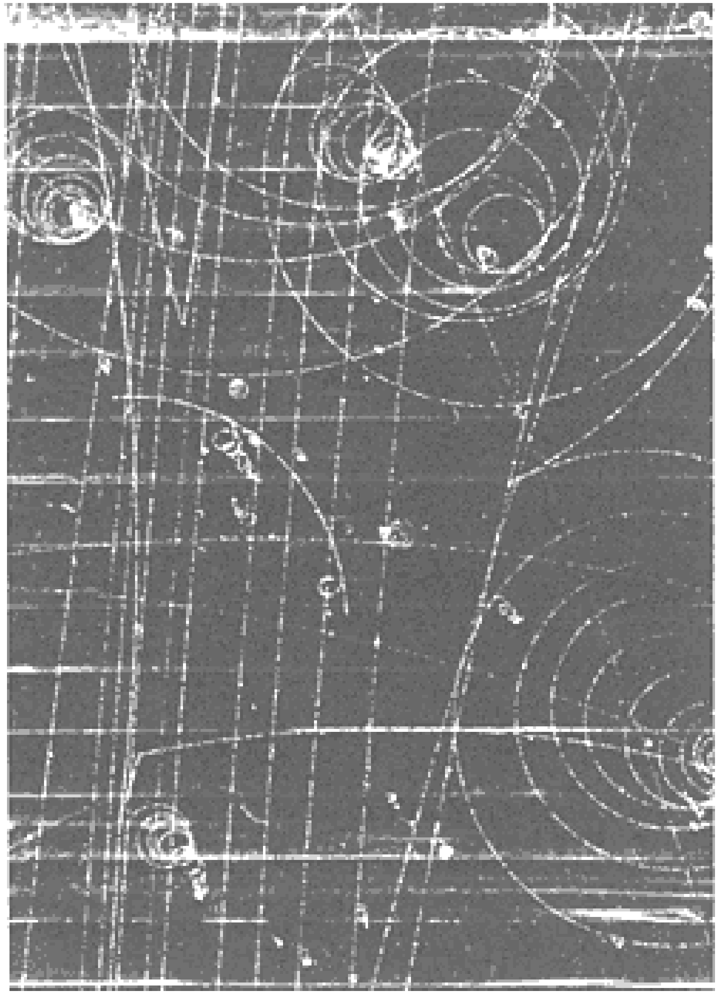


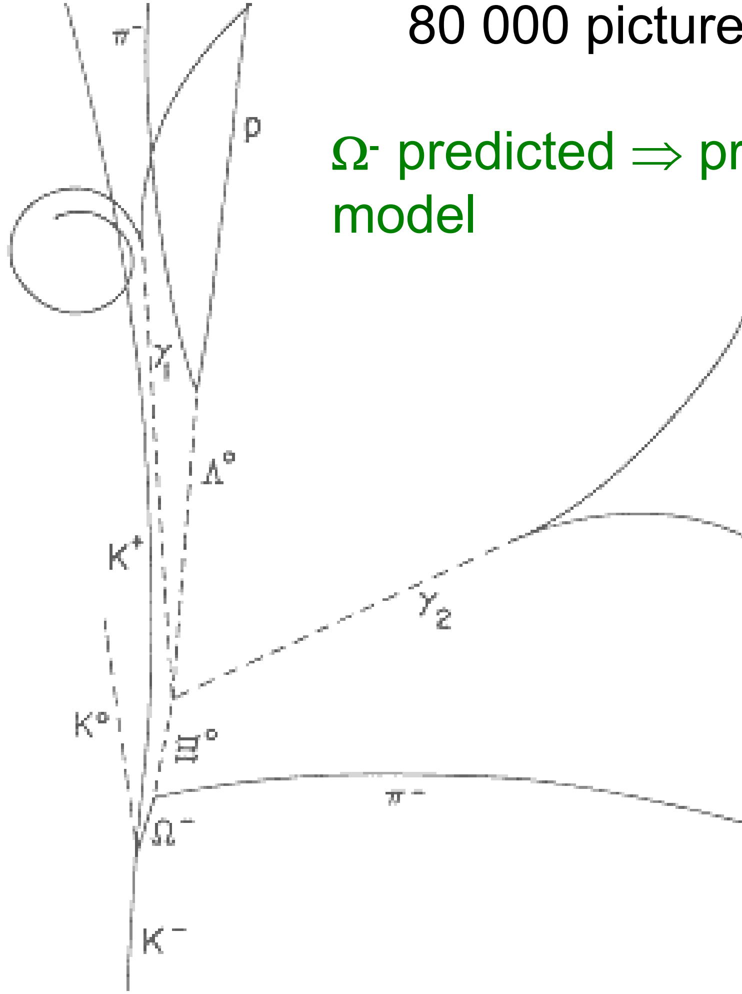
baryons $3/2^+$

Ω^- predicted



February 1964 : the Ω^- discovery





Brookhaven bubble chamber ; 80 000 pictures

Ω^- predicted $\Rightarrow$ probe of the SU(3) model

$$K^-p \rightarrow \Omega^- K^+ K^0$$

$$\Omega^- \rightarrow \Xi^0 \pi^-$$

$$\Xi^0 \rightarrow \Lambda \pi^0$$

$$\Lambda \rightarrow p\pi^-$$

But in order for the flavour-SU(3) symmetry to be «exact», all the particles of a given SU(3) multiplet should have the same mass

Masses for particles in few multiplets:

$\pi^\pm$	0^-	140 MeV	p	$\frac{1}{2}^+$	938 MeV
π^0	0^-	135	n	$\frac{1}{2}^+$	940
$K^\pm$	0^-	494	Λ	$\frac{1}{2}^+$	1160
K^0, $\overline{K}^0$	0^-	498	Σ^+	$\frac{1}{2}^+$	1189
η	0^-	549	Σ^0	$\frac{1}{2}^+$	1192
η'	0^-	958	Σ^-	$\frac{1}{2}^+$	1197
$\rho^\pm$, ρ^0	1^-	770	Ξ^0	$\frac{1}{2}^+$	1315
ω	1^-	783	Ξ^-	$\frac{1}{2}^+$	1321
K^*	1^-	892	Ω	$3/2^+$	1672
ϕ	1^-	1020			

It is not the case….

Schematic view of the symmetries

$$H_{had} = H_{0,Strong} + \delta H_{Strong} + \delta H_{elm}$$

baryons, $J^P \neq$
mesons, $J'^{P'}$

strangeness

Electric charge !

<u>generators</u>

Isospin is conserved by strong interaction

$[H_{0,Strong} + \delta H_{Strong}, \lambda_i] = 0$

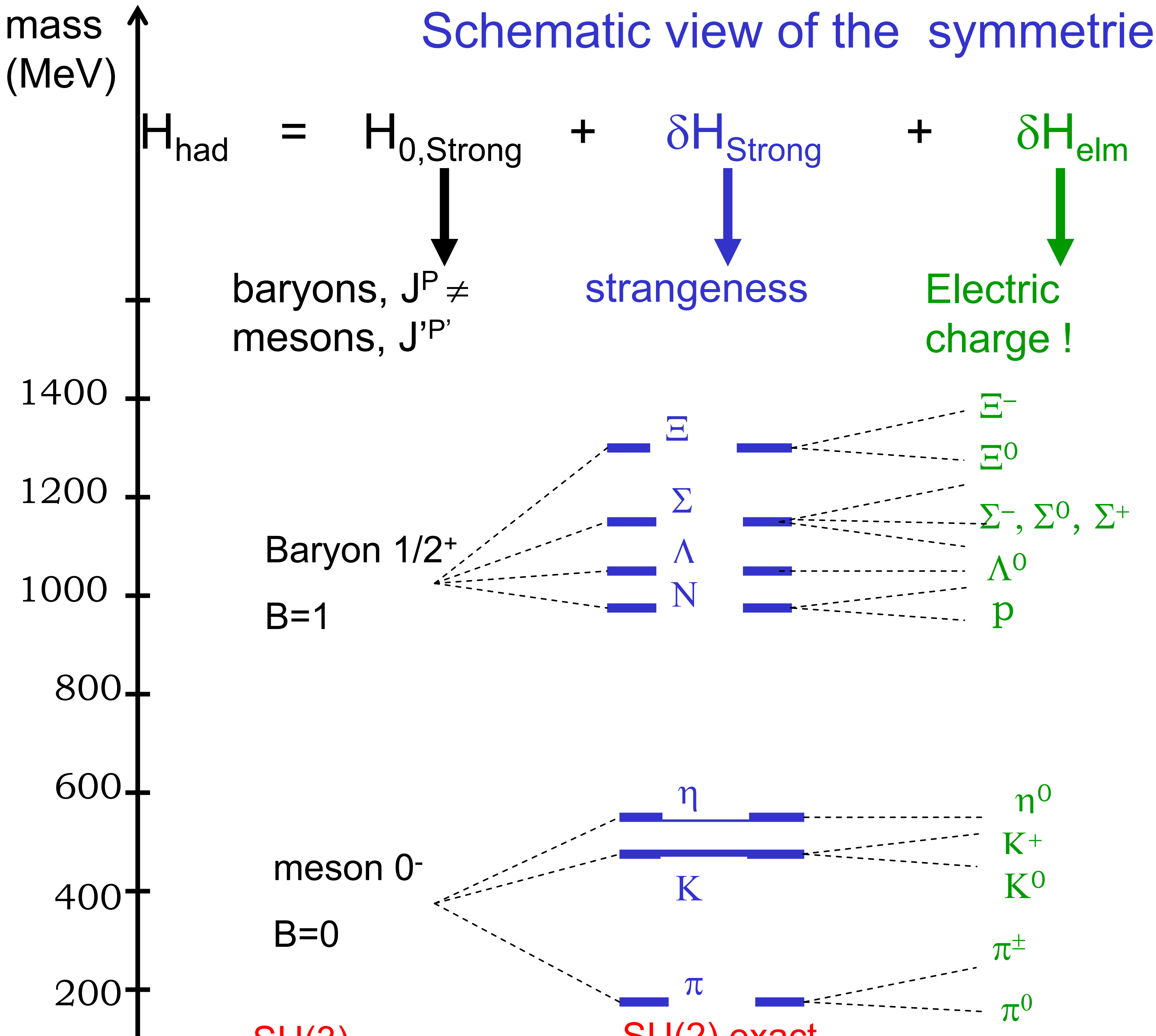
$i = 1,2,3,8$

the Strangeness is not conserved by δH_{Strong}

$[\delta H_{Strong}, \lambda_i] \neq 0$

but $[H_{o,Strong}, \lambda_i] = 0$

$i = 4,5,6,7$



SU(3) exact

MH SU(2) exact
SU(3) broken

ch SU(2) broken

Summary: strangeness, flavour-SU(3)

- The introduction of a new quantum number S (strangeness) allows to explain few experimental facts
- Change in the charge/isospin relation to take into account the strangeness operator : $Q = I_3 + B/2 + S/2$
- Widening of the isospin formalism : flavour-SU(3) symmetry (8 generators λ_i)
- Ordering of the mesons and baryons in SU(3) multiplets

  (singlet, octet or decuplet)
- This model has predicted new particles (η, Ω) which have been found !
- But this symmetry is not exact (it is a broken symmetry).

Can we go further?

- Looking at : $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$

  $$3 \otimes \bar{3} = 1 \oplus 8$$

  can suggest that the states corresponding to the fundamental representations of SU(3) are the hadrons components

  Historical trial : {p,n} fundamental representation of SU(2) …it didn't work !

  (the pions are not made of protons and neutrons, the Δ neither)

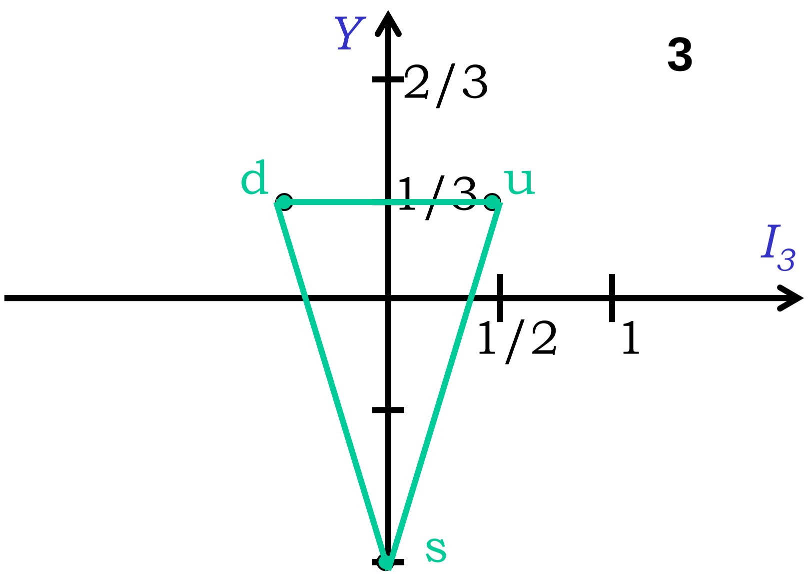
3. Quarks

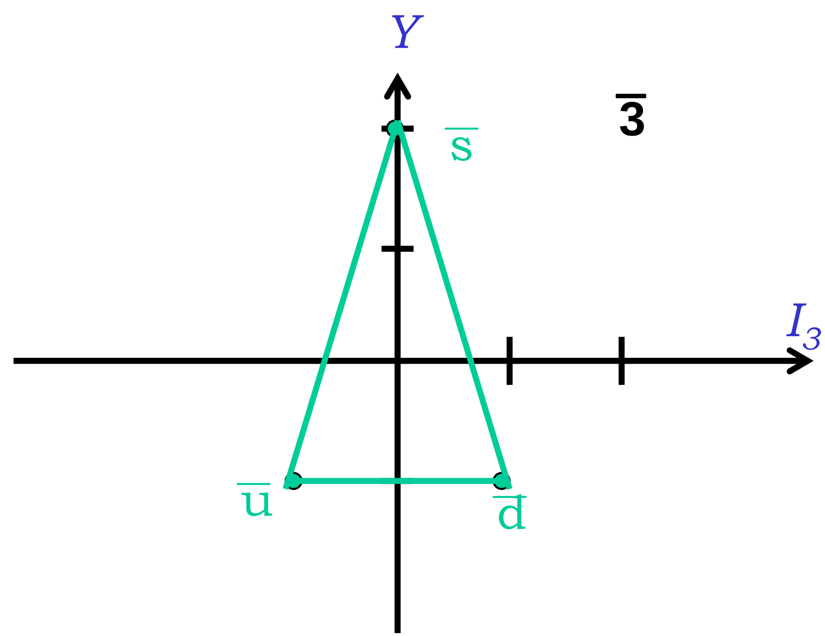
$$|u\rangle,\ |d\rangle,\ |s\rangle \qquad 3$$

$$|\bar{u}\rangle,\ |\bar{d}\rangle,\ |\bar{s}\rangle \qquad \bar{3}$$

Fundamental representations

We have created 3 monsters !





Electric charge:

Q(u)=2/3

Q(d,s)=-1/3

Baryonic number:

B(u,d,s)=1/3

Isospin:

I_3(u,d)=±1/2

I_3(s)=0

Strangeness:

S(u,d)=0

S(s)=-1

3.1 Multiplets construction for the mesons :

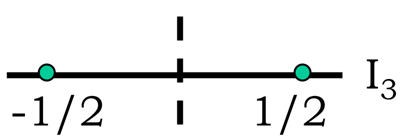
- The mesons are made of $q_1\overline{q}_2$
- If one only has *u* and *d* quarks, one needs to combine : $2 \otimes \underbrace{\overline{2}}_{=2} = 1 \oplus 3$

  (we are only interested in the isospin)
- One gets:

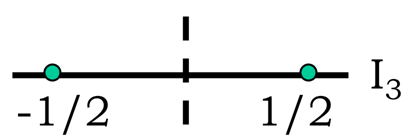
$$
\begin{cases}
|I=1, I_3=1\rangle = u\overline{d} \\
|I=1, I_3=0\rangle = \sqrt{1/2}\left(u\overline{u} - d\overline{d}\right) \\
|I=1, I_3=-1\rangle = d\overline{u} \\
|I=0, I_3=0\rangle = \sqrt{1/2}\left(u\overline{u} + d\overline{d}\right)
\end{cases}
$$

- Graphically:

Quarks multiplet

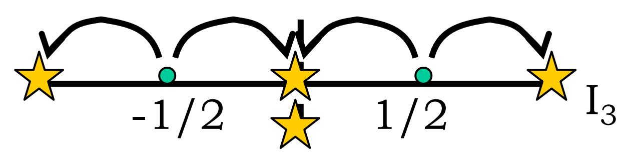


anti-quarks mutiplet



One gets:

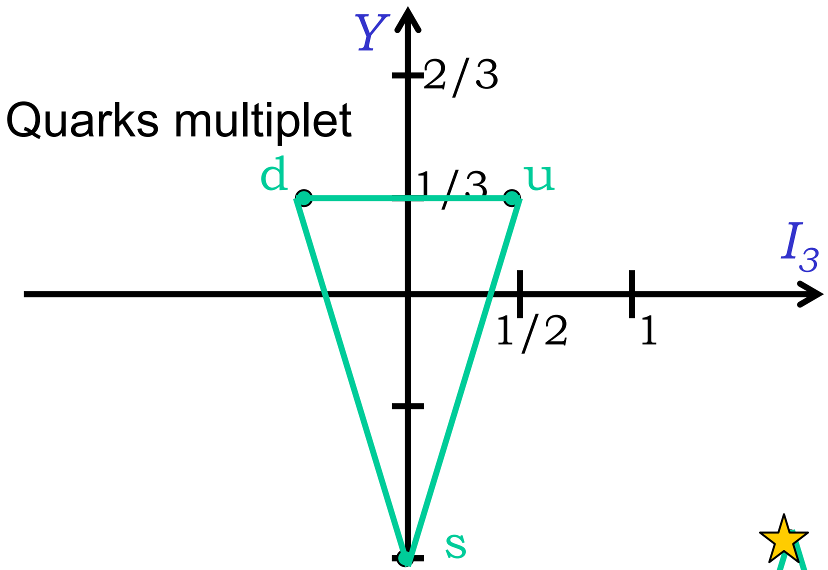
1 symmetric state,
3 anti-symmetric states

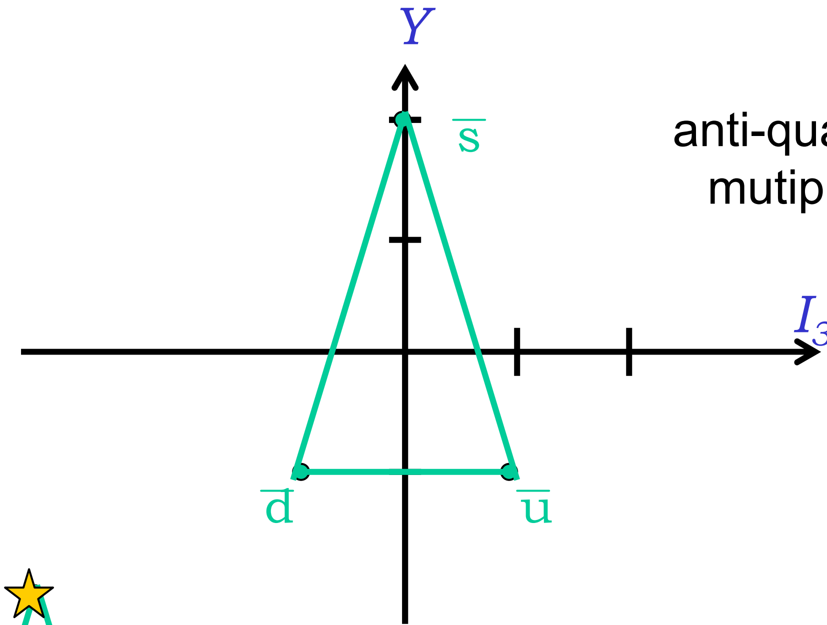


1(S)+3(A)

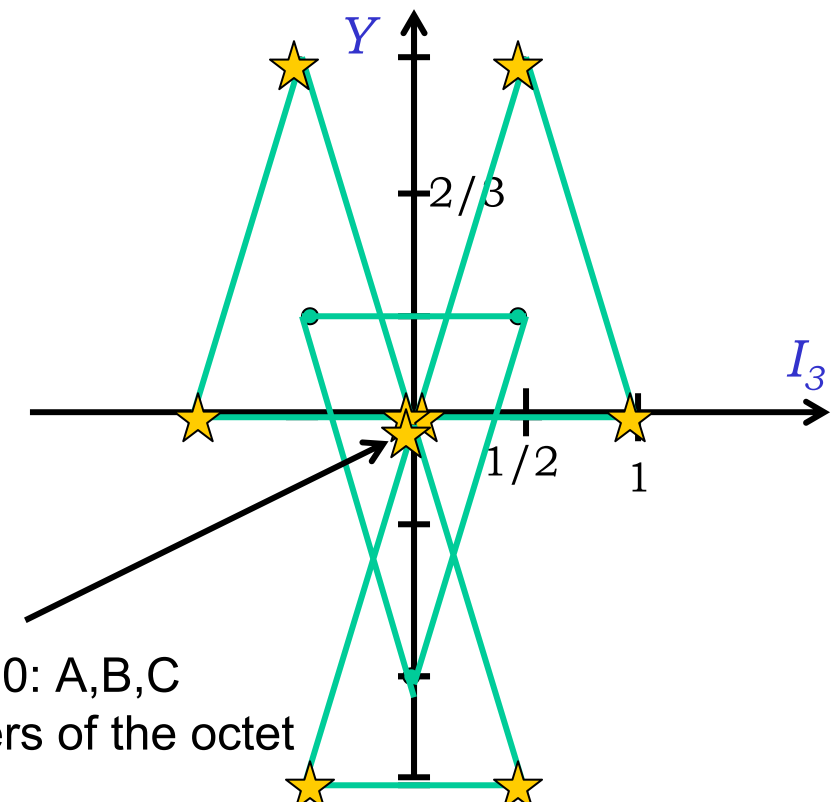
This diagram is obtained superposing the center of gravity of anti-quark multiplet on each quark position

With u,d,s : one needs to take into account the hypercharge Y





anti-quarks
mutiplet



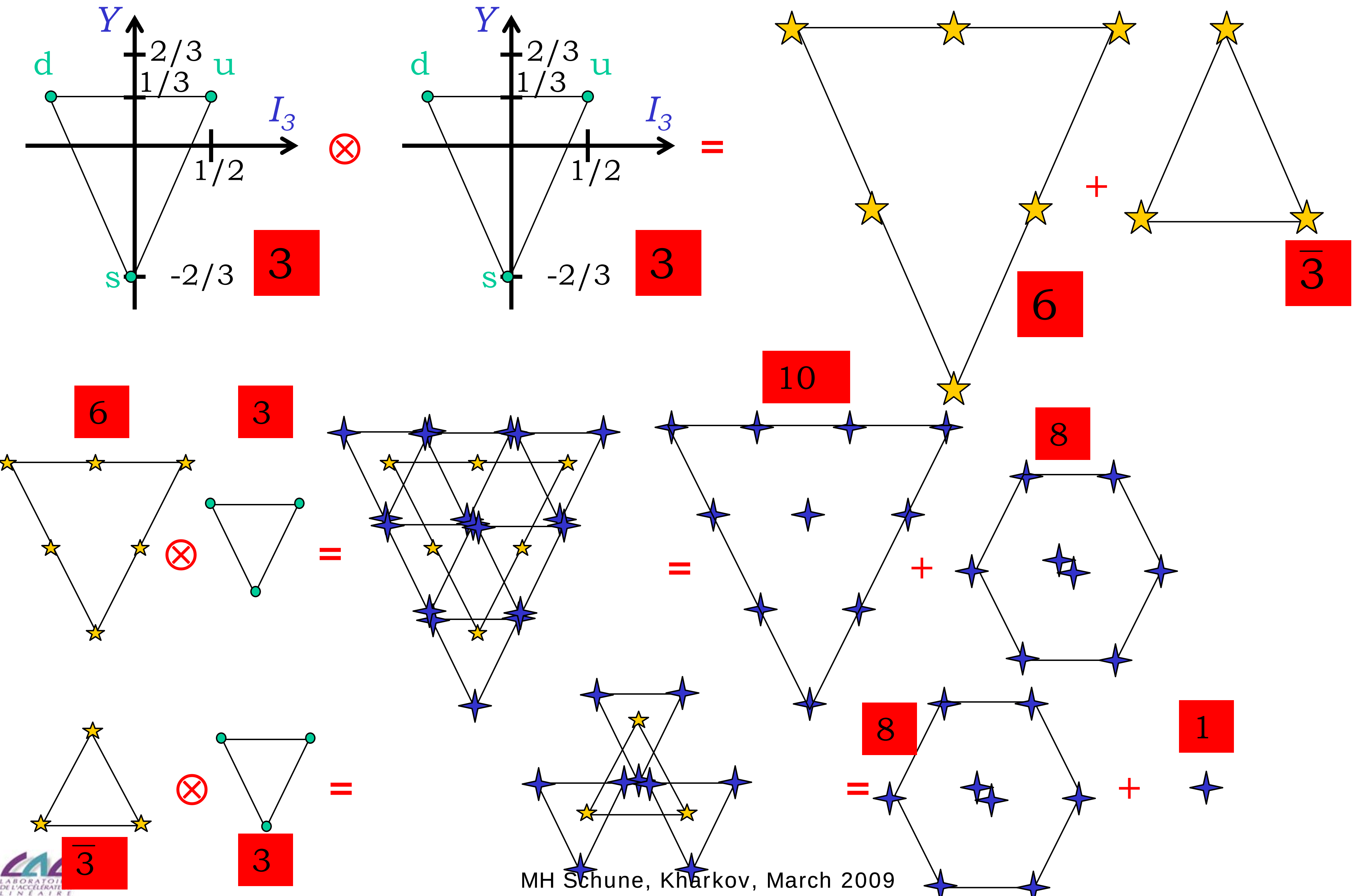
$3 \otimes \bar{3}$

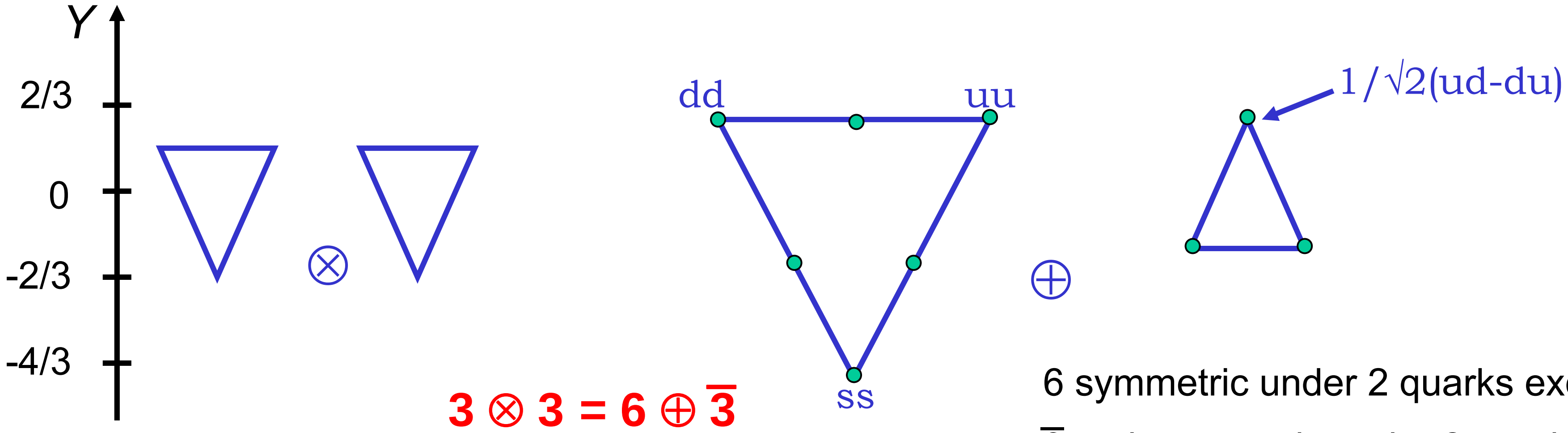
=

$1 \oplus 8$

3 states I_3=0, Y=0: A,B,C
One singlet, 2 members of the octet

3.2 Multiplets construction for the baryons : 3⊗3⊗3



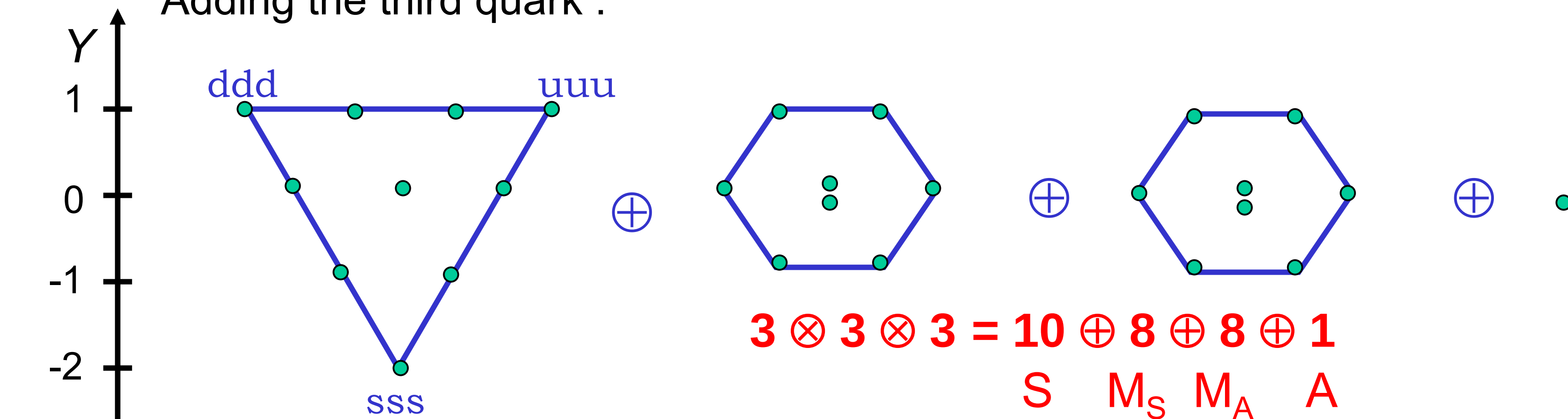


$$\mathbf{3 \otimes 3 = 6 \oplus \overline{3}}$$

6 symmetric under 2 quarks exchange

$\overline{3}$ anti-symmetric under 2 quarks exchange

Adding the third quark :



$$\mathbf{3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1}$$

S M_S M_A A

M_S and M_A indicate symmetry (S) and anti-symmetry (A) by exchange of the first 2 quarks

For the spin:
$2 \otimes 2 \otimes 2$
$= 4 \oplus 2 \oplus 2$
S M_S M_A

One can create (flavour-SU(3),SU(2)-spin) categories : for example :

S: (10,4) + (8,2), M_S : (10,2) + (8,4) + (8,2) + (1,2), A: (1,4) + (8,2)

One finds back the $3/2^+$ baryons (S (10,4)), $1/2^+$ (S (8,2)) but things are getting complicated !

3.3 Quarks masses :

SU(3) mutiplets built with quarks as elementary components of the hadrons. Can their mass be estimated ?

1) Mass difference:

Assumption : the mass of a hadron is given by the sum of the masses of its components.

- For the $3/2^+$ decuplet :

$$\underset{sss}{M_\Omega} - \underset{dss}{M_{\Xi^*}} = \underset{dss}{M_{\Xi^*}} - \underset{uus}{M_{\Sigma^*}} = \underset{uus}{M_{\Sigma^*}} - \underset{udd}{M_\Delta} = m_s - m_{u,d}$$

- For the $1/2^+$ octet:

$$\underset{dss}{M_\Xi} - \underset{uds}{M_\Sigma} = \underset{dss}{M_\Xi} - \underset{uds}{M_\Lambda} = \underset{uds}{M_\Lambda} - \underset{udd}{M_N} = m_s - m_{u,d}$$

- these differences lead to :

  142, 145, 153 MeV

  123, 202, 177 MeV

  Averaging by multiplet

- Quite different …
- Leaving apart those differences, one could say that the quark mass difference is :

$$m_s - m_{u,d} \sim 160 \text{ MeV}$$

m_u, m_d difference :

$$\begin{array}{cccccc} uud & dds & uss & uus & dss & udd \end{array}$$

$$M_p + M_{\Sigma^-} + M_{\Xi^0} = M_{\Sigma^+} + M_{\Xi^-} + M_n$$

$$\left(M_p - M_n\right) = \left(M_{\Sigma^+} - M_{\Sigma^-}\right) + \left(M_{\Xi^-} - M_{\Xi^0}\right)$$

-1.3 MeV -8 MeV +6.4 MeV

-1.6 MeV

$\Rightarrow m_d > m_u$ and $m_d \approx m_u$ + 2 MeV

One should take into account that the masses we are talking about are effective masses of bound states (the hadrons). They are sometimes called constituents masses (they are not the ones appearing in the Lagrangian)

2) Using $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ states:

$$M_\omega \sim M_\rho \left(u\bar{u}\right) \sim 780 \text{ MeV}$$
$$M_\phi \quad \left(s\bar{s}\right) \sim 1.02 \text{ GeV}$$
$$M_\psi \quad \left(c\bar{c}\right) \sim 3.1 \text{ GeV}$$
$$M_{\Upsilon(4S)} \quad \left(b\bar{b}\right) \sim 10.6 \text{ GeV}$$

$$\Rightarrow \begin{cases} m_u \sim m_d \sim 390 & \text{MeV} \\ m_s \sim 510 & \text{MeV} \\ m_c \sim 1.6 & \text{GeV} \\ m_b \sim 5 & \text{GeV} \end{cases}$$

3) By measuring the magnetic moment

- Measurement of the proton intrinsic magnetic moment : $\mu_p = g\mu_B = 2.7928456(11)\,\frac{e}{2m_p}$

$\Rightarrow$ the proton is not a point like particle (otherwise one would get : g =2) $\qquad \vec{\mu} = g\mu_B \vec{S}$

- Predictions for the proton and neutron intrinsic magnetic moments using the spin of the quarks :

p = (uud): *uu*: S=1, *d*: S=1/2, and the proton has : S=1/2 and S_z = +1/2

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\,|1,1\rangle\,\left|\frac{1}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\,|1,0\rangle\,\left|\frac{1}{2},+\frac{1}{2}\right\rangle \quad + \text{ permutations}$$

$$|u\uparrow u\uparrow \quad d\downarrow\rangle \qquad |u\uparrow u\downarrow \quad d\uparrow\rangle$$

one gets:

$$+\mu_u+\mu_u-\mu_d \qquad +\mu_u-\mu_u+\mu_d$$

$$\mu_p = \frac{1}{3}(4\mu_u - \mu_d)$$

$$\mu_n = \frac{1}{3}(4\mu_d - \mu_u)$$

if m_u=m_d *and* q_u=-2q_d *(same g):* μ_u=-2 μ_d

$$\boxed{\frac{\mu_n}{\mu_p} = -\frac{2}{3} \quad \text{Quarks model prediction}}$$

Experimentally one measures: μ_n/μ_p = -0.68497945(580) !!

$$\boxed{\text{The quarks exist and one has : } m_u \approx m_d \text{ et } q_u=-2q_d}$$

- One can also derive the *u,d* quarks mass from these predictions :

$$\mu_u = \frac{2}{3}\mu_p = \frac{2}{3}2.79\frac{e}{2m_p} \Rightarrow m_u = \frac{m_p}{2.79} = 340\ \text{MeV}$$

- Similarly the *s* quark mass can be estimated from the measurement of the Λ magnetic moment :

$$\mu_s = \mu_\Lambda = -0.6\frac{e}{2m_p} \Rightarrow m_s = 510\ \text{MeV}$$

Quark model prediction

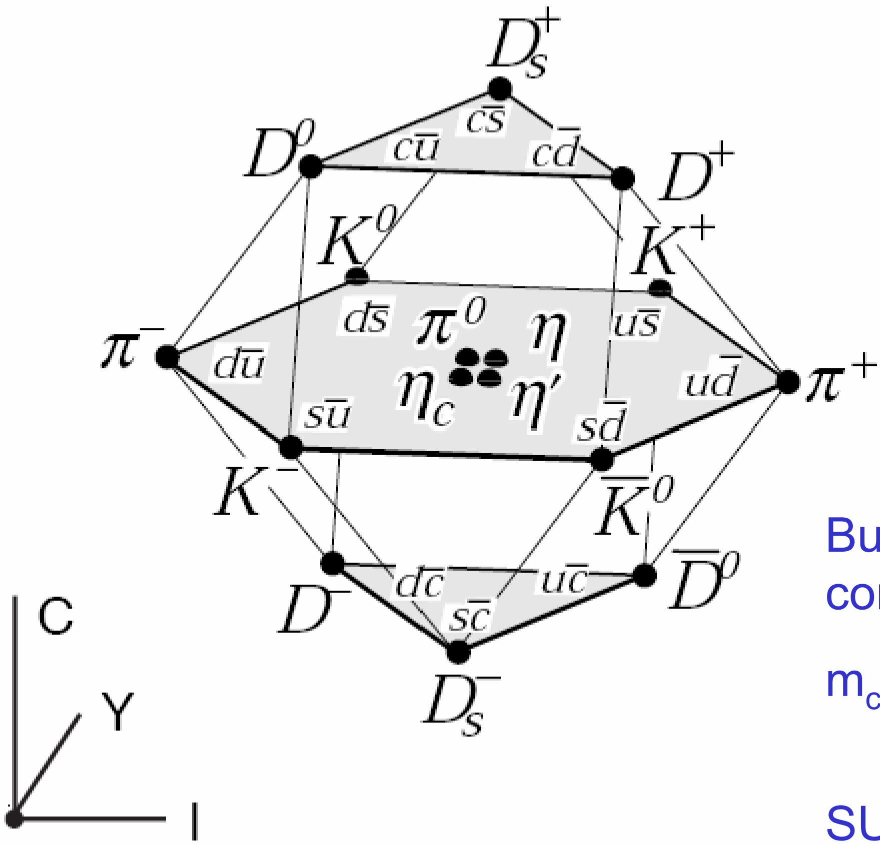
measurement

- These determinations of the *u,d* and *s* quarks masses are in agreement with the ones obtained using the $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ states

$\Rightarrow m_u \sim m_d$ and $m_s > m_u$, m_d

This « explains» why SU(2) was a more exact symmetry than SU(3).

One can even go furtther and add a 4th quark : the charm quark $\Rightarrow$SU(4)



But things are getting more and more complicated and the symmetry is much worse :

$m_c >> m_s > m_u \sim m_d$

SU(2) : is a very good symmetry

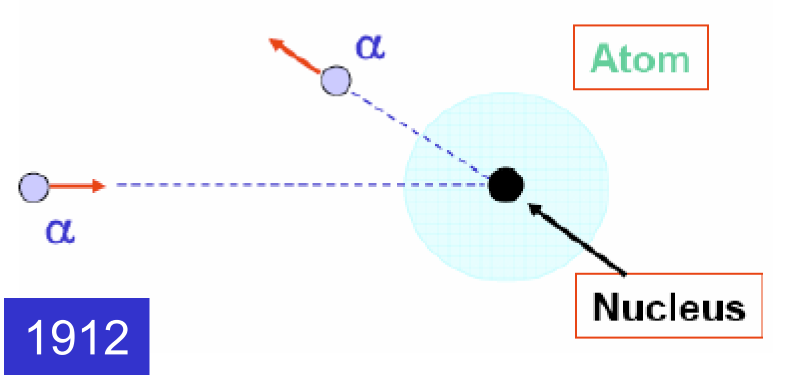
SU(3) is still reasonable ~30%

SU(4) does not work well

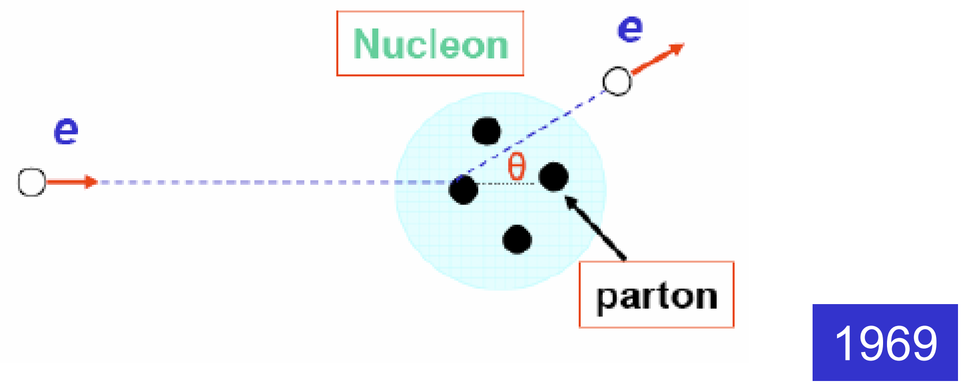
3.4 Experimental search for the hadrons internal structure :

- It is very similar to Rutherford's experiment :

  A particle (which is assumed to be point-like) is sent on a composite object

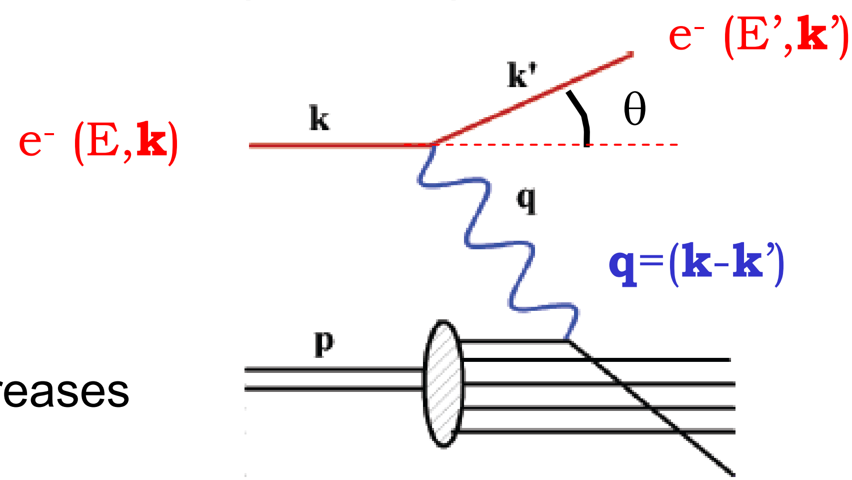


Discovery of the nucleus inside an atom



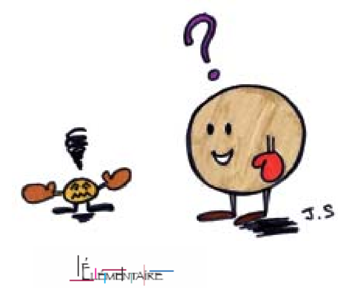
Discovery of the point-like spin 1/2 partons

elastic scattering $e^-p \to e^-p$ the e^- is a point-like particle, what about the proton ?



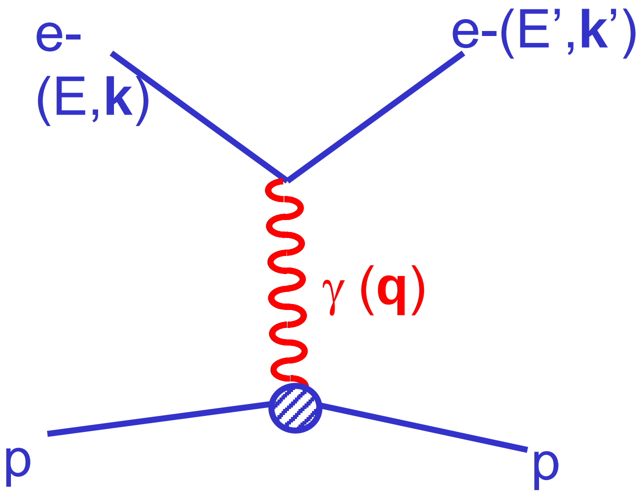
$$\lambda = \frac{2\pi\hbar}{|\mathbf{k}|}$$

When E increases, λ decreases



λ is the distance explored by the electron in the proton

- More precisely :



$$\vec{q} = \vec{k} - \vec{k}' \qquad q^0 = E - E'$$

$$Q^2 = -q^2$$

It is the γ which explores the proton

- One has :

$$\lambda = \frac{2\pi\hbar}{|q|} \simeq \frac{2\pi\hbar}{2\sqrt{EE'}\sin(\theta/2)}$$

$$-q^2 = \left(\vec{k} - \vec{k}'\right)^2 - \left(E - E'\right)^2 = -2m^2 - 2kk'\cos\theta + 2EE'$$

$$\approx 2EE'(1-\cos\theta) = 4EE'\sin^2(\theta/2) = s\sin^2(\theta/2)$$

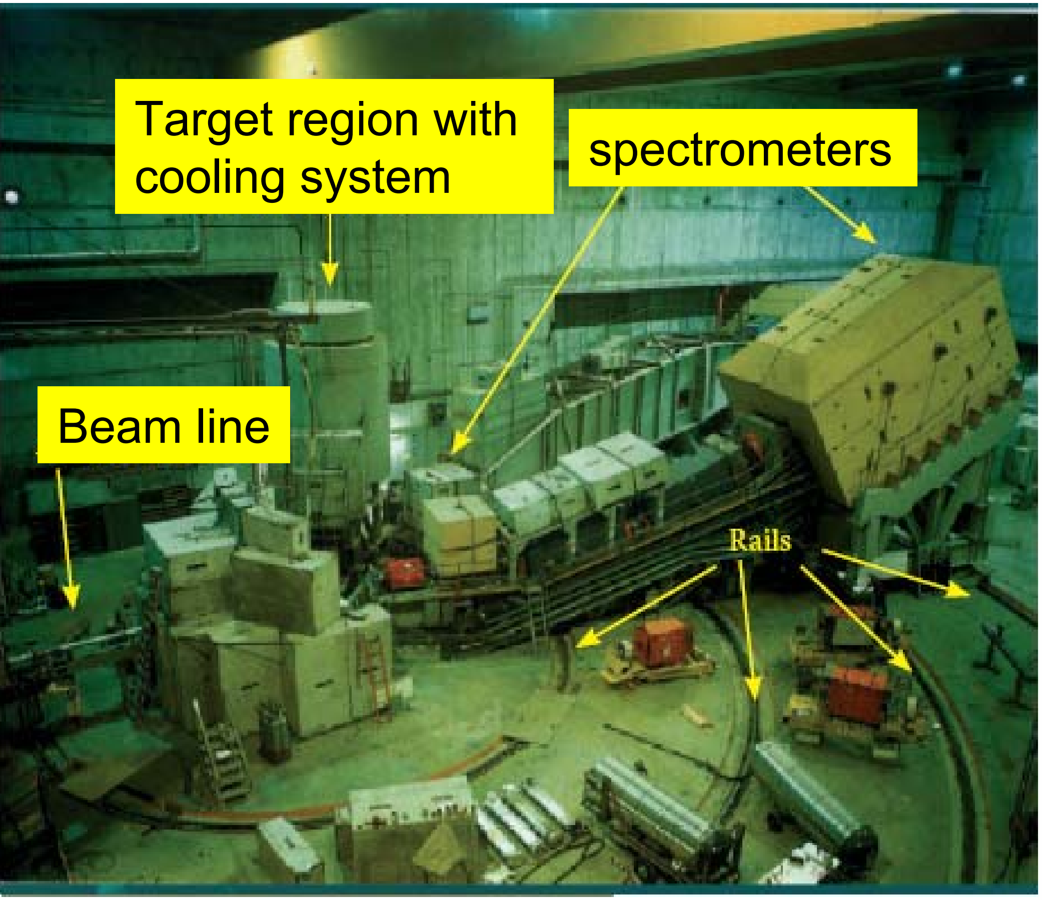
In the e-p rest frame

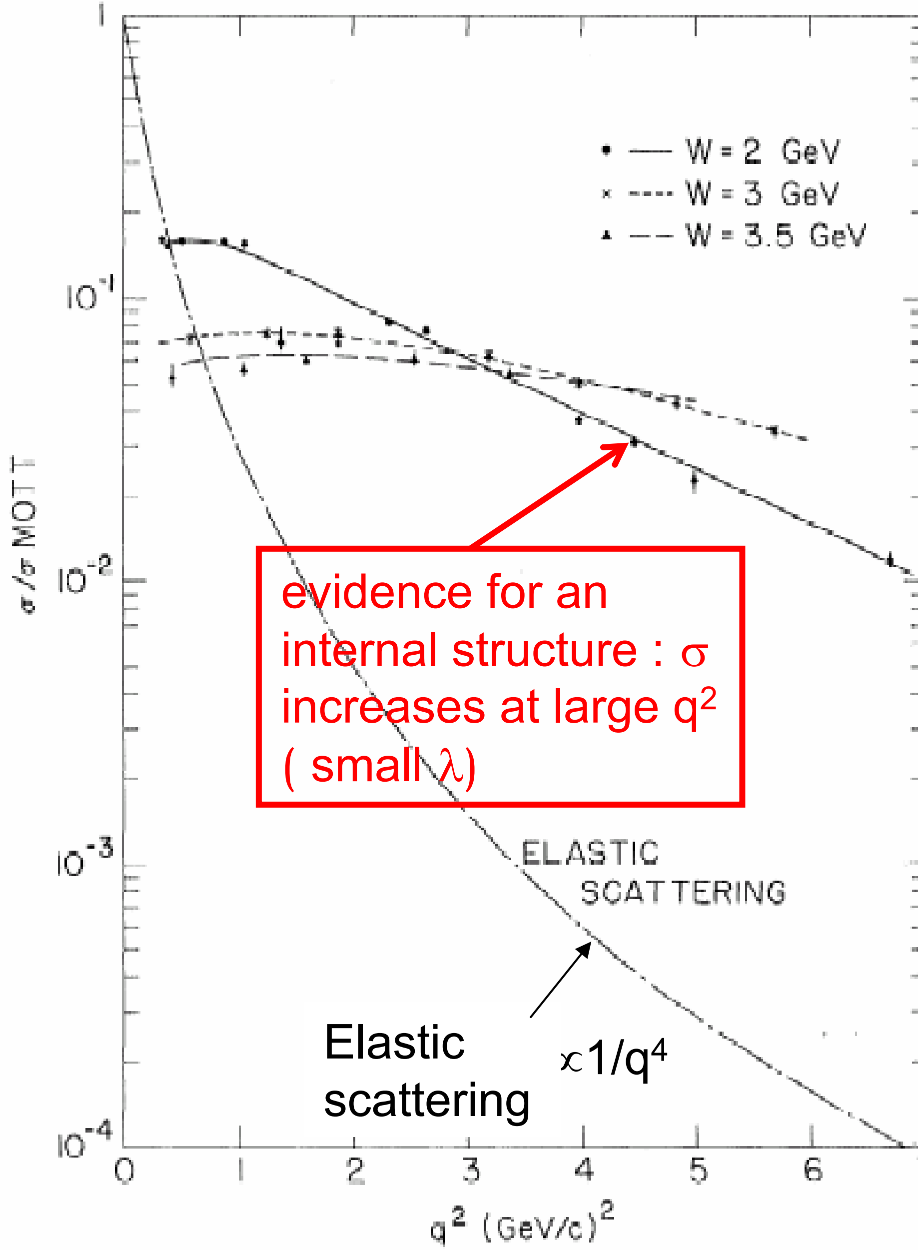
At a given energy, λ decreases if θ increases.

The collisions taking place at large angles explore the proton internal structure.

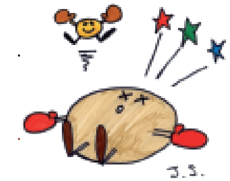
(even if the energy increases, if θ~0, we do not explore the proton internal structure)

SLAC 1969

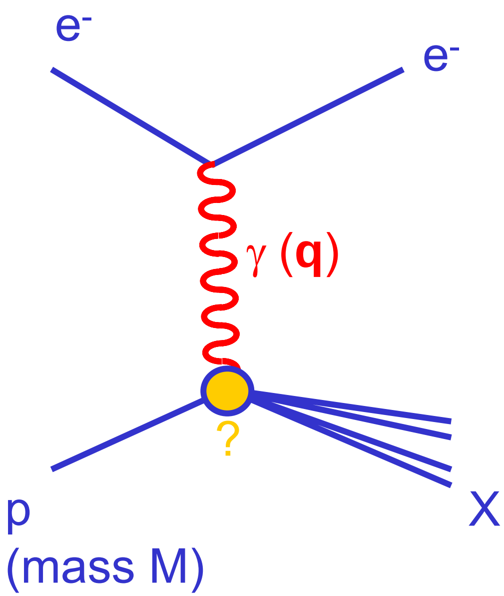

Target region with cooling system
spectrometers
Beam line
Rails


W = 2 GeV
W = 3 GeV
W = 3.5 GeV
σ / σ MOTT
q² (GeV/c)²
evidence for an internal structure : σ increases at large q² (small λ)
ELASTIC SCATTERING
Elastic scattering
∝1/q⁴

The energy involved in the process is such (20 GeV electrons) that the proton is "destroyed" : Deep Inelastic Scattering : new particles are created.



$e^- p \to e^- X \quad (X = p, n, \pi, ..)$



$$Q^2 = -q^2$$

$$x = \frac{Q^2}{2M(E - E')}$$

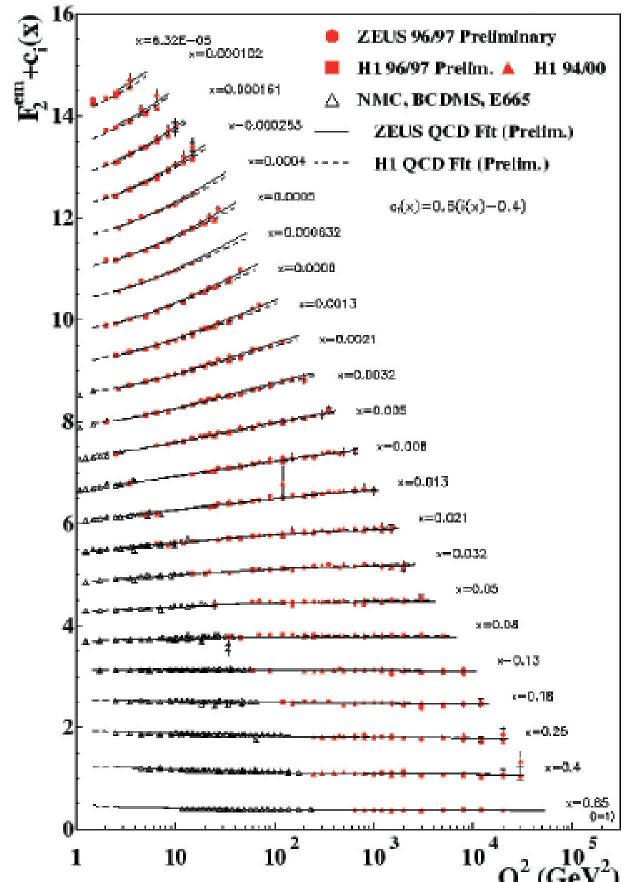
$$W^2 = M_X^2 = (p_p + q)^2$$

2 variables are needed to describe the interaction : Q^2 and x

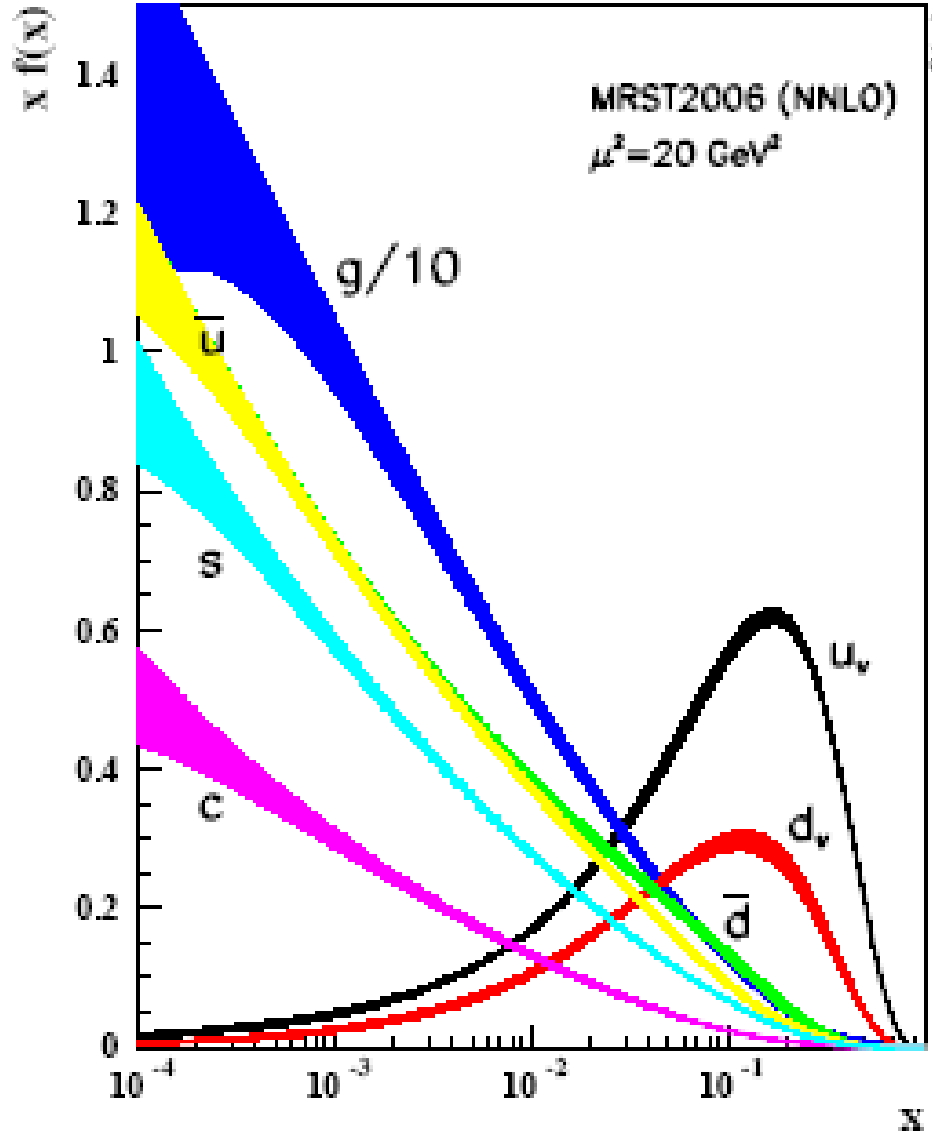
x : fraction of the momentum of the proton carried by the quark involved in the reaction

➡ Structure function of the proton

For collisions on point-like objects : horizontal line (x>0.1)



Many different experiments and ways of measuring what is inside the ptoton (energy, polarization …) over 40 years



Most of the energy is carried by u and d quarks but … not only : gluon, sea quarks …

Summary: quarks, flavour-SU(3)

- The states of the fundamental representations of SU(3) : 3 et $\bar{3}$ can be interpreted as the elementary components of the hadrons : the quarks.
- 3 quarks: u, d, s (non integer charges !)
- One can build the mesons and baryons multiplets in SU(3) using a graphical method
- One can compute their masses (constituent mass).
- Experimentally one has shown that quarks are real and are the internal components of the hadrons :

  - measurement of the intrinsic magnetic moment of the proton

  - elastic and inelastic scatterings

But it is not the end of the story …

4. colour-SU(3) : QCD

4.1 the colour :

- The symmetry of the fundamental states is problematic in some cases. For example :

$$\Delta^{++} \quad J^P = \frac{3}{2}^+ \qquad u\uparrow u\uparrow u\uparrow$$

is described by a symmetric wave function.

However … the Pauli's exclusion principle tells us that the wave function should be anti-symmetric when exchanging identical fermions

(the combination of 3 identical fermions is forbidden)

In addition, why in Nature do we have only

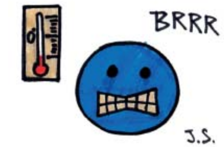
$$q_1 q_2 q_3 \ , \quad \overline{q}_1 \overline{q}_2 \overline{q}_3 \ , \quad q_1 \overline{q}_2$$

And not $q_1 q_2 \ , \quad \overline{q}_1 \overline{q}_2$

?

- We make the hypothesis for a number quantum number :

the colour



Quarks have a colour-charge:

Red , Blue or Green

In this case the Δ^{++} is:

$$u_R \uparrow u_B \uparrow u_G \uparrow$$

The problem has disappeared : the quarks are not anymore identical

But how to describe the proton ?

$$u_R u_B d_G,\ u_R u_G d_B,\ u_B u_G d_B,\ \ldots.$$

Hypothesis : all observed particles are « colourless » (combination of the 3 colours, or colour-anticolour) :

$$p = RGB$$
$$\overline{p} = \overline{R}\,\overline{G}\,\overline{B}$$
$$\pi = R\overline{R} + G\overline{G} + B\overline{B}$$

- A *qq* state carries colour :

  $q_R q_G,\ q_R q_B$, …. $\Rightarrow$ *qq* does not exist

- 3 colours $\Rightarrow$ symmetry colour-SU(3)

$$\psi = \psi(x)\chi_{spin}\chi_{colour}$$ *colour* =R,G,B

- By analogy with flavour-SU(3) (same group) : one can define in an *ad hoc* way that the colour states correspond to a pair of values (Y^C, I_3^C).

One writes :

	Quarks		Anti-quarks	
	I_3^C	Y^C	I_3^C	Y^C
R	1/2	1/3	-1/2	-1/3
G	-1/2	1/3	1/2	-1/3
B	0	-2/3	0	2/3

- One can write the confinement hypothesis (the fact that we do not see coloured particle) as :

  $Y^C = I_3^C = 0$

- What are the consequences of this hypothesis ?

  Let's take the particle $q^m \bar{q}^n$ made of *m* quarks and *n* anti-quark.

  restricting to $B>0 \Rightarrow m > n$.

  In terms of colour: $r^{\alpha} g^{\beta} b^{\gamma} \bar{r}^{\bar{\alpha}} \bar{g}^{\bar{\beta}} \bar{b}^{\bar{\gamma}}$ (r^{α} means that there is α red quarks)

And one can write : $m = \alpha + \beta + \gamma \quad > \quad n = \bar{\alpha} + \bar{\beta} + \bar{\gamma}$

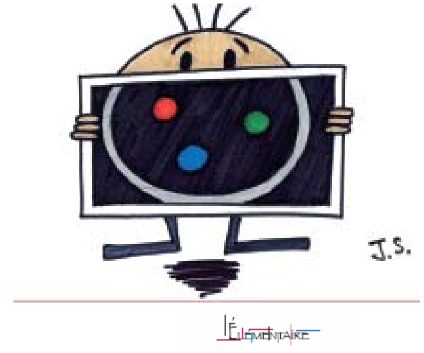
- Using the values of Y^C and I_3^C , the condition $Y^C = I_3^C = 0$ is written as :

$$I_3^c = \frac{(\alpha - \bar{\alpha})}{2} - \frac{(\beta - \bar{\beta})}{2} = 0 \qquad \text{for the } q^m q^n \text{ particle}$$

$$Y^c = \frac{(\alpha - \bar{\alpha})}{3} + \frac{(\beta - \bar{\beta})}{3} - \frac{2(\gamma - \bar{\gamma})}{3} = 0$$

$$\Rightarrow \alpha - \bar{\alpha} = \beta - \bar{\beta} = \gamma - \bar{\gamma} \equiv p \qquad (p \geq 0)$$

$$m - n = 3p$$



The only allowed combinations are thus :

$$q^{3p+n}\bar{q}^{n} = q^{3p}q^{n}\bar{q}^{n} = (3q)^{p}(q\bar{q})^{n}$$

The states can be classified as :

Allowed : $qqq, \quad q\bar{q}$

Forbidden by confinement : $qq, \quad qq\bar{q}, \quad qqqq$

Non integer charge hadrons

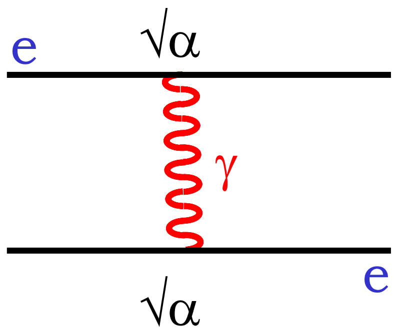
Possible states but non observed $qq\bar{q}\bar{q}, \quad qqqq\bar{q}$

Open question

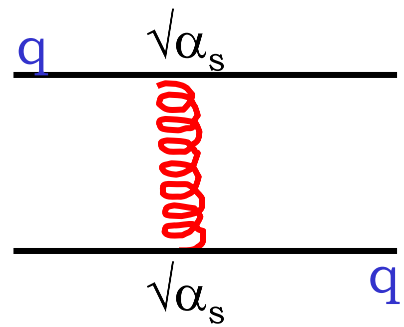
4.2 colour-SU(3), gluons

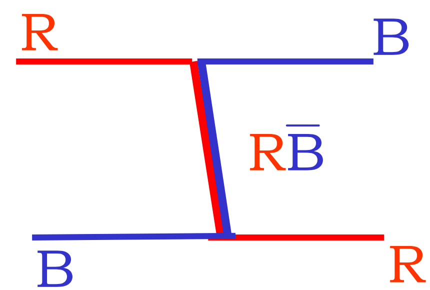
- Charge for the strong interaction : colour charge
- SU(3): 3^2-1=8 generators ⇒ 8 gluons : vector particles of the strong interaction
- quarks carry a colour (R,G ou B).
- The colour exchange takes place through 8 «bi-coloured» gluons

EM:

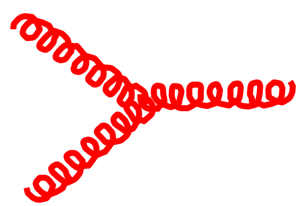


strong:



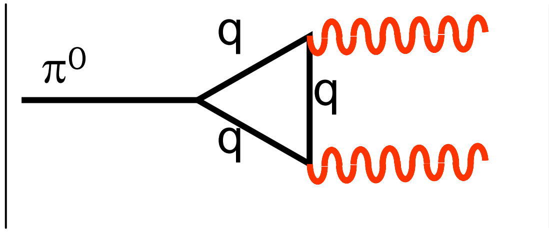


- Main difference between the electromagnetic and strong interactions : the gluons are coloured so they can interact with each other, ≠ of the photon (SU(3) is a non abelian group):



Experimental evidence for colour

- 1) $\pi^0 \to \gamma\gamma$:



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To obtain the experimental value of $\Gamma(\pi^0 \to \gamma\gamma)$ $\sim |\alpha N_c|^2 \sim 9\alpha^2$ $\Rightarrow N_c=3$

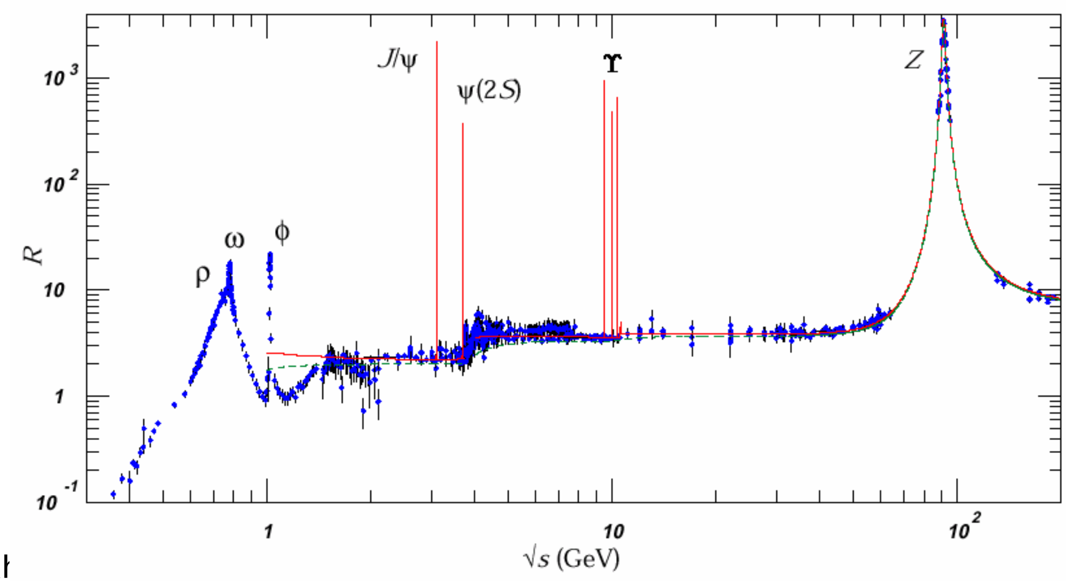
measurement

- 2) One of the historical reason to rescue the Pauli's exclusion principle

$$\Delta^{++} \qquad |u_R \uparrow u_B \uparrow u_G \uparrow\rangle \qquad J^p = 3/2^+$$

- 3) The R ratio

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_C \sum_i q_i^2 = N_C\left[\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] = N_C \frac{11}{9}$$

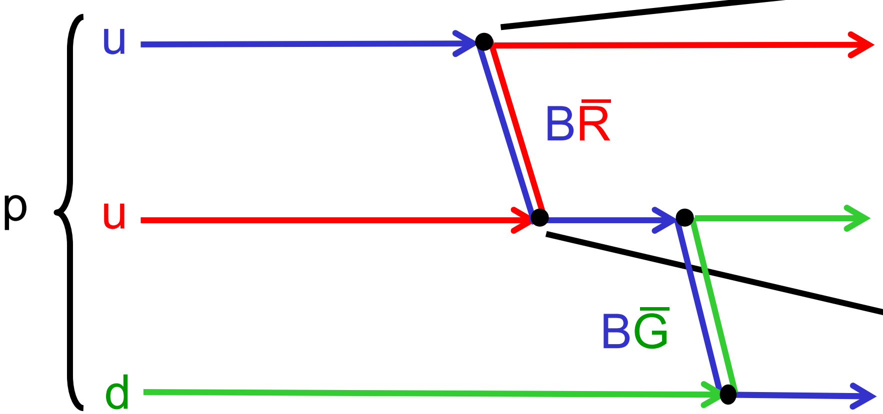


The colour, which has been thought about because of the Δ^{++} puzzle does exist and $N_c=3$!!

4.3 QCD (Quantum ChromoDynamics):

QCD is the theory based on colour-SU(3) which describes the strong interaction (non broken local gauge symmetry).

Proton description :

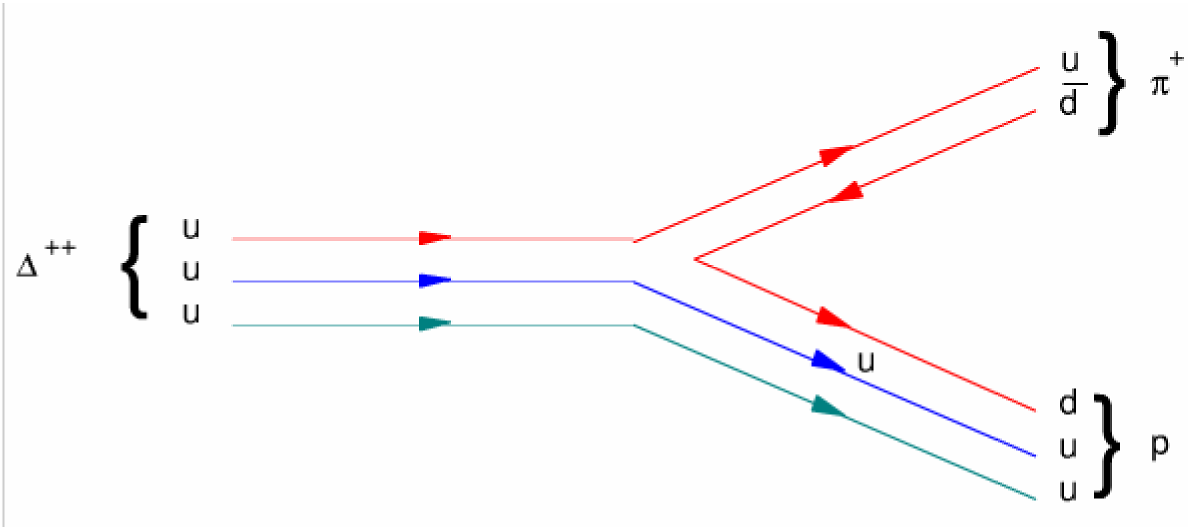


At this vertex the $B\bar{R}$ gluon transforms the $B \rightarrow R$ (removing the B, leaving the $\bar{R}$ which is equilibrated by the R)

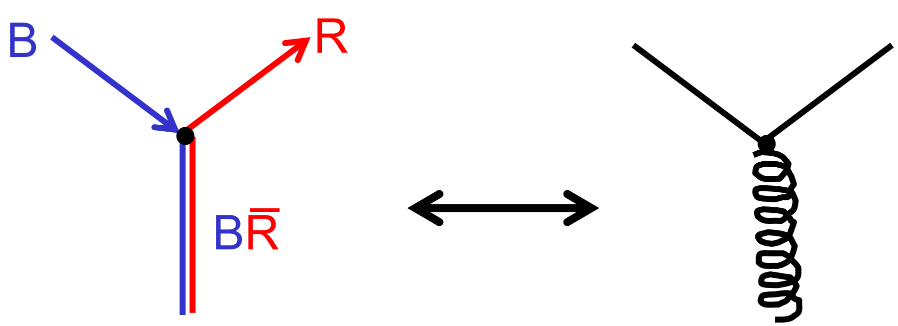
$R\,\bar{R}$ annihilate, the B remains. $R \rightarrow B$

The proton is a mixture of : $u_R u_B d_G$, $u_R u_G d_B$, $u_B u_G d_R$, ….

Quark diagram taking into account the color charge : example of the $\Delta \to p\pi$ decay

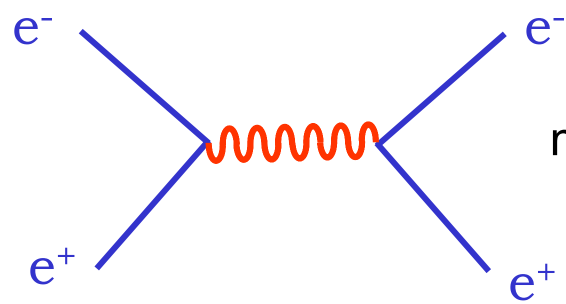


What differs wrt electromagnetism : the gluon is coloured (the g is neutral)



Colour charge dispersion

QED : interaction between 2 electrons, α_{EM} running



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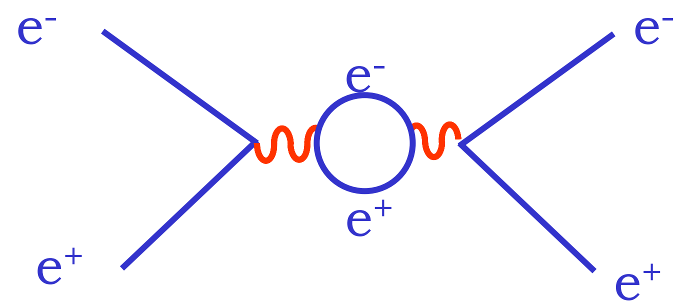

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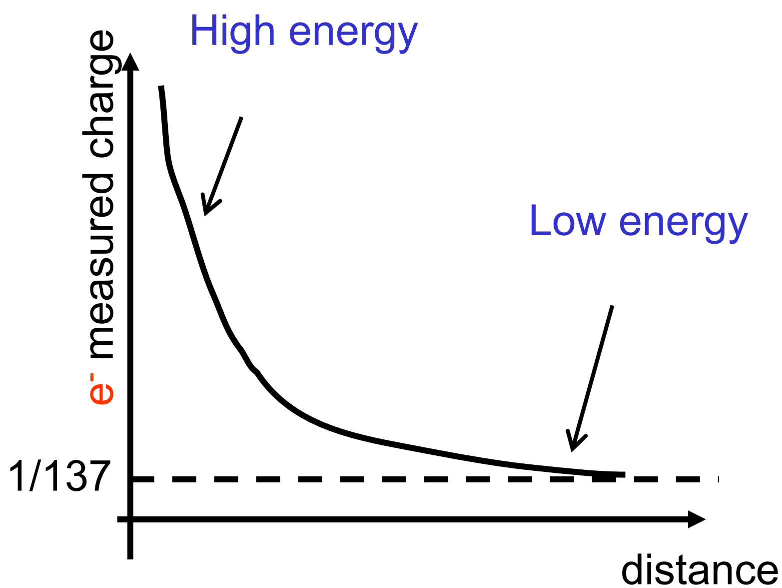
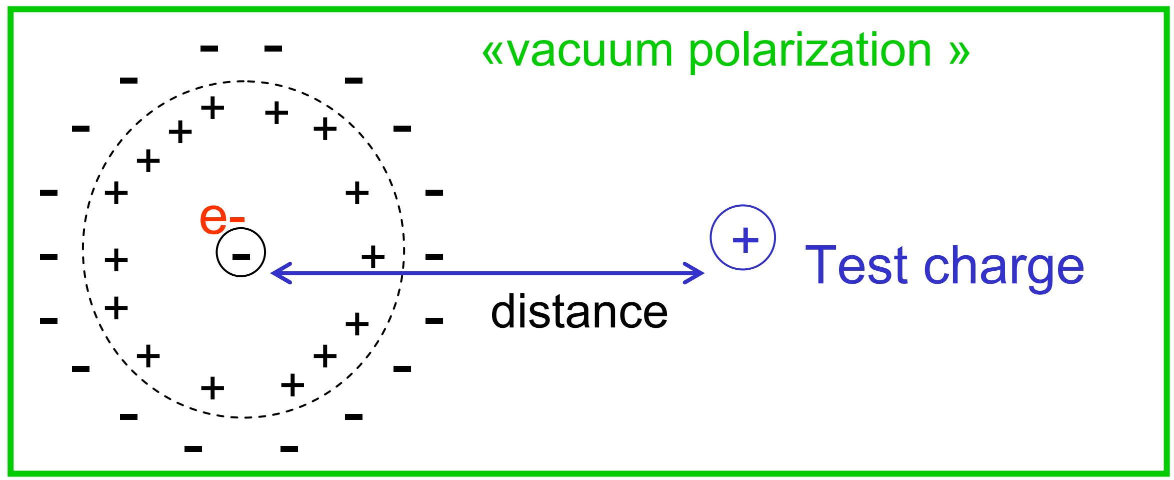


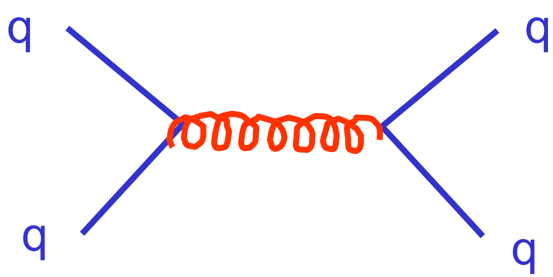


The charge « e» measured experimentally differs from e_0 which appears at the lowest order of the feynman graphs. At low energy $e^2/4\pi=1/137$ but the interaction modifies the charge

$$\alpha\left(Q^2\right)=\frac{\alpha\left(\mu^2\right)}{1-\frac{\alpha\left(\mu^2\right)}{3\pi}\log\left(\frac{Q^2}{\mu^2}\right)}$$

μ and Q : energy scales

QCD : interaction between 2 quarks, α_S running



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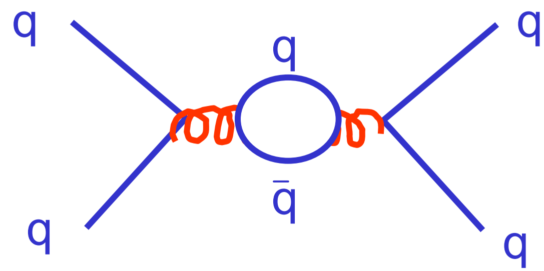

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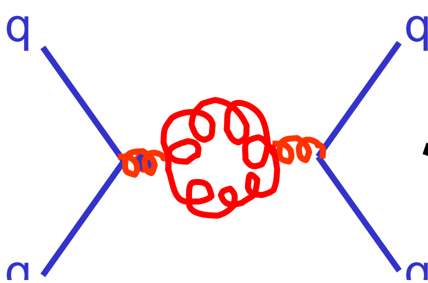
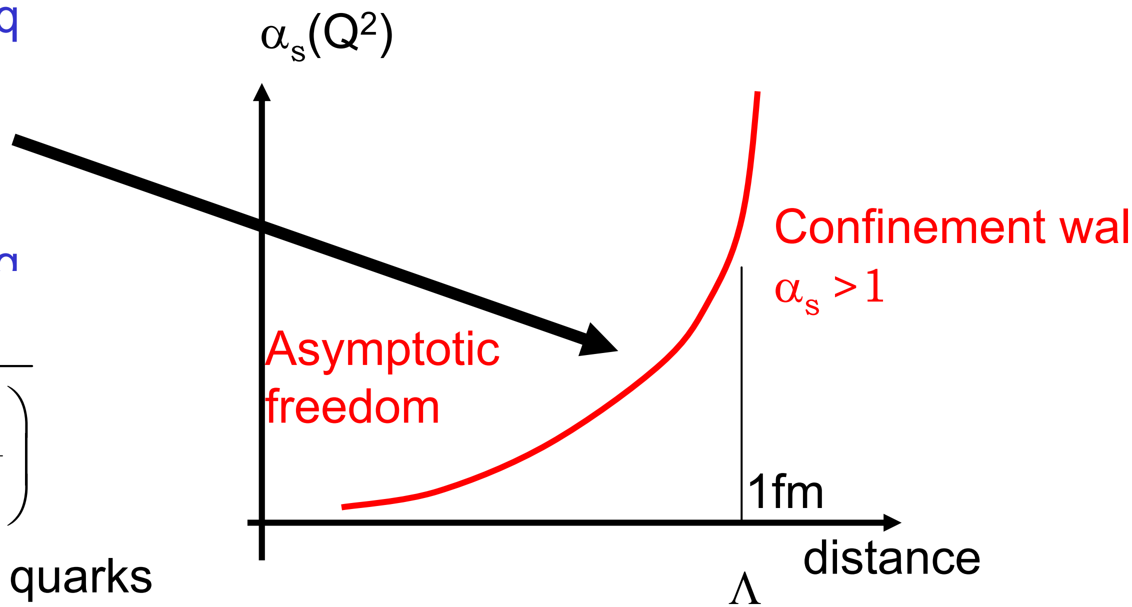
but also by :





$$\alpha_s\left(Q^2\right) = \frac{\alpha_s\left(\mu^2\right)}{1 + \frac{\alpha_s\left(\mu^2\right)}{12\pi}\left(11n - 2f\right)\log\left(\frac{Q^2}{\mu^2}\right)}$$

nb of colours   nb of quarks

If f <17: when $Q^2\uparrow$, $\alpha_s\downarrow$ (for n=3).

In the limit $\mu \rightarrow \Lambda$, $\alpha_S(\mu^2) \rightarrow \infty$:

$$\alpha_s\left(Q^2\right) = \frac{12\pi}{\left(11n - 2f\right)\log\left(\frac{Q^2}{\Lambda^2}\right)}$$

(Λ~100 MeV)

~~$Q^2 \sim \Lambda^2$ strong coupling perturbations~~

$Q^2 \gg \Lambda^2$ weak coupling perturbations

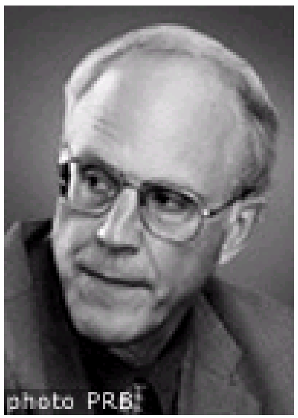
non intuitive !

The α_S running (main property of QCD) was analytically computed in june 1973.

The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"

$$\alpha_s\left(Q^2\right) = \frac{\alpha_s\left(\mu^2\right)}{1+\frac{\alpha_s\left(\mu^2\right)}{12\pi}\left(11n-2f\right)\log\left(\frac{Q^2}{\mu^2}\right)}$$



David J. Gross

1/3 of the prize

USA

Kavli Institute for Theoretical Physics, University of California
Santa Barbara, CA, USA

b. 1941

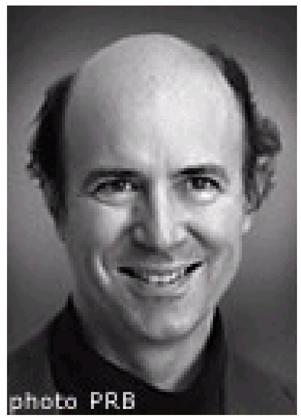
H. David Politzer

1/3 of the prize

USA

California Institute of Technology
Pasadena, CA, USA

b. 1949



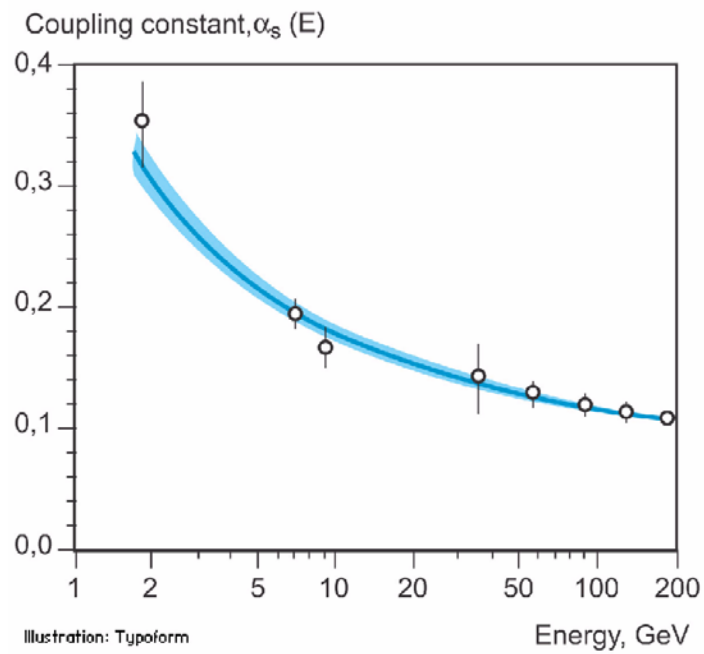
Frank Wilczek

1/3 of the prize

USA

Massachusetts Institute of Technology (MIT)
Cambridge, MA, USA

b. 1951



Coming back to the hadrons masses

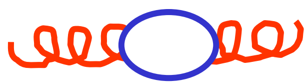
<u>QCD computation of the mesons mass</u>

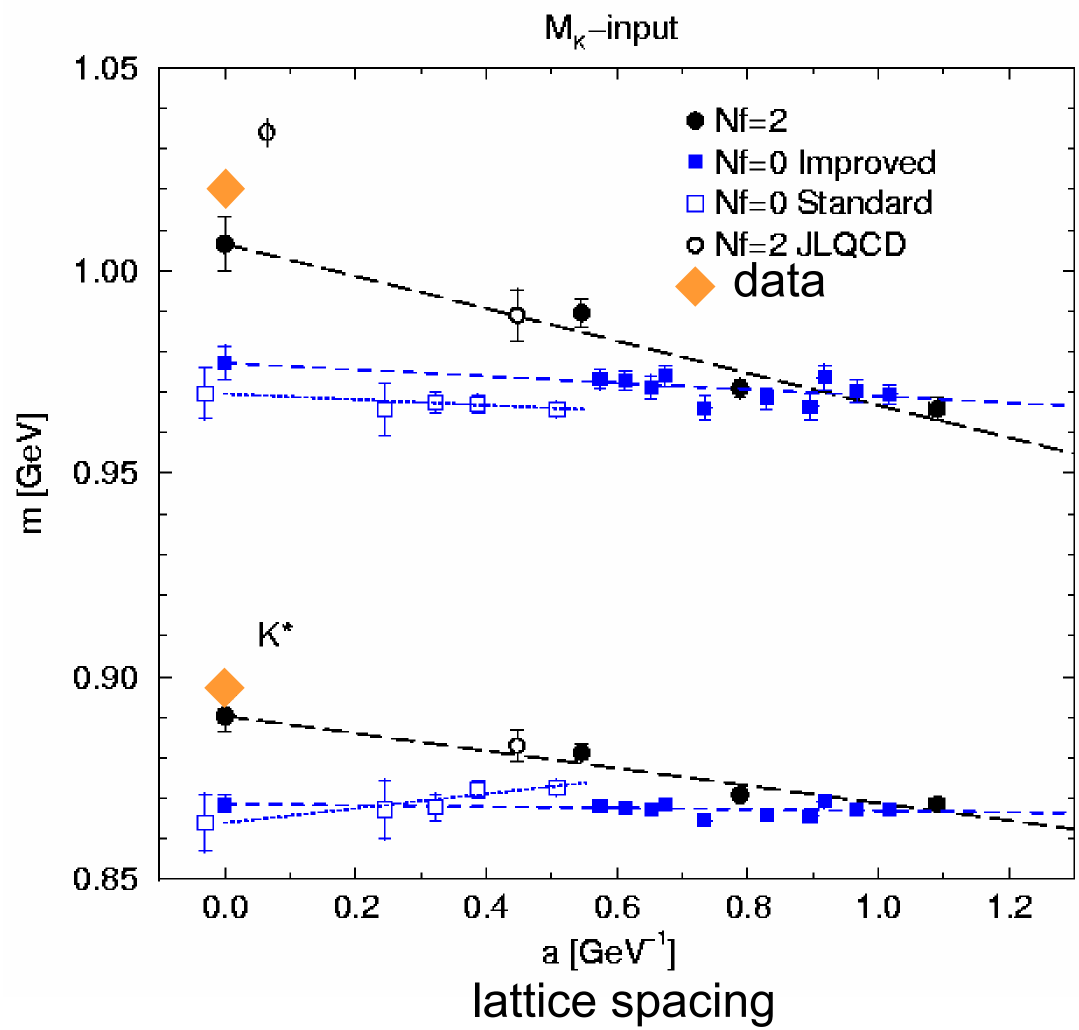
(numerical computation : lattice QCD)

In the example : ϕ and K*

- computation without $g \to qq$
- computations with $g \to qq$
- data

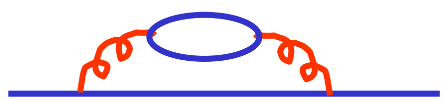
$\Rightarrow$ the contributions are important





observed mass:

m_i + interaction =



in Lagrangian

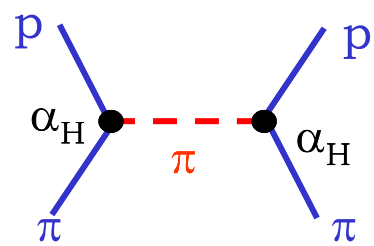
ex: $m_u + m_u + m_d \sim 5\%\ m_{proton}$!

In order to obtain the physical mass, one needs to introduce the vacuum polarization in the computation (non perturbative domain $\Rightarrow$ one cannot perform perturbation series computations for QCD, models are needed !)

QCD and other models

- A model for the nucleus cohesion : force represented by the π exchange with a short range potential.

  It works at low energy.. where QCD does not work because $\alpha_s > 1$; ad-hoc models are the only way out :



At low energy the πp interaction is only sensitive to the colour charge which is at the surface of the nucleon (in other words at the distance of ~1 fm ($\hbar c/m_\pi$) It is a non perturbative regime

α_H is not a fundamental constant but an epiphenomenom due to the π and p structure but also to α_S which is a fundamental constant.

- Model for the hadron cohesion : a potential of shape *V=-1/r + kr*

but all that is coming from the strong interaction ???

yes but since we do not know how to perform the calculations we should use modes… which differ for different scales.

It is somewhat similar to the Van der Walls force for which the potential is in $1/r^6$ even if they come from the electromagnetic interaction which is in $1/r$

Summary : QCD, colour-SU(3)

- Need for an additional quantum number : the colour

  for which we have indirect experimental proofs

- colour-SU(3) : exact local gauge symmetry (QCD)

  3 colours

  8 gluons carrying colours (self-interaction) they mediate the strong interaction

Only « white" particles can be seen (only qqq and $q\bar{q}$)

- The « running” of the coupling constant α_s differs from the running in QED

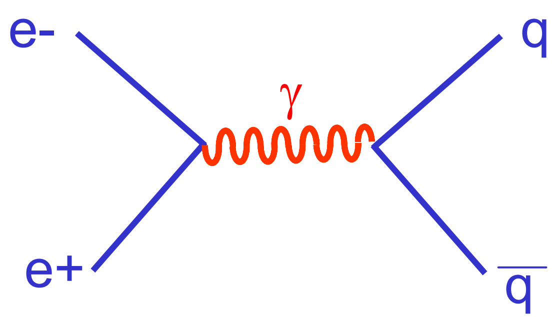
  $\alpha_s \searrow$ when E $\nearrow$: asymptotic freedom

  $\alpha_s \nearrow$ when E $\searrow$: quarks confinement

  $\Rightarrow$ low energy : non perturbative regime, cannot perform series expansion , models are needed

5. Hadronization

- $e^+e^- \to q\bar{q}$



but the quarks cannot be seen since they carry colour

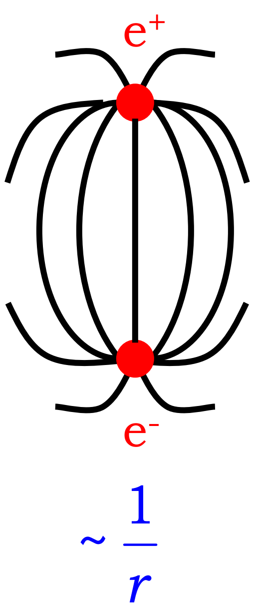
- Strong interaction becomes too « strong » when the distance between the 2 quarks is larger than 1 fm ; the perturbative computations are impossible

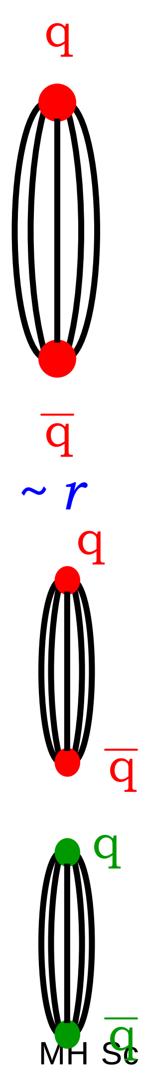
Models to describe the transition between quarks and hadrons

= hadronization models

Hadronization model

- The strong interaction :
  - is similar to the electromagnetic interaction at short distances
  - becomes stronger when the distance increases





Model: quarks undergo the potential

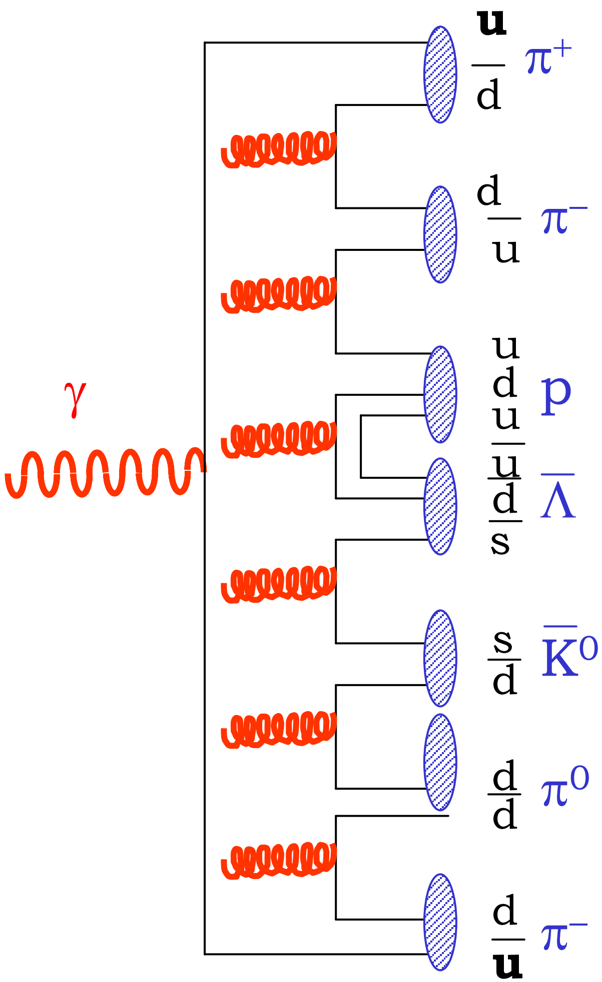
$$V(r) = \frac{\alpha_S}{r} + ar$$

If the energy in the tube is constant per unit of length, $V\uparrow$ when $r\uparrow$

If one tries to separate the $q\bar{q}$ pair, V increases enough to create a new $q\bar{q}$ pair

Quarks are confined

The hadronization phenomenon predicts the existence of jets of particles

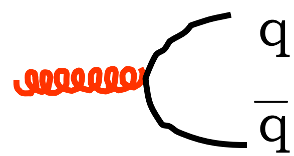


<u>Example:</u>

An uu pair is produced

$\rightarrow$ as soon as the (coloured) quarks try to move away the force gets stronger

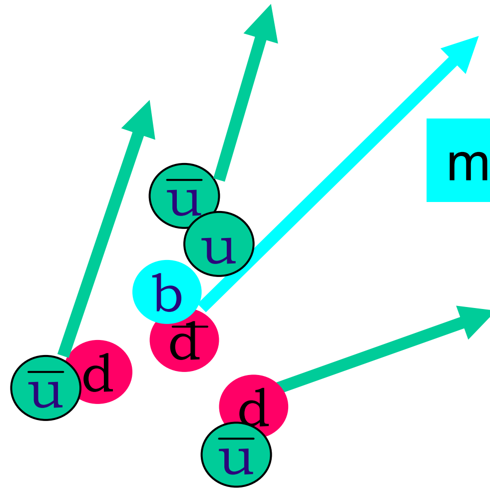
$\rightarrow$the stored energy materializes as



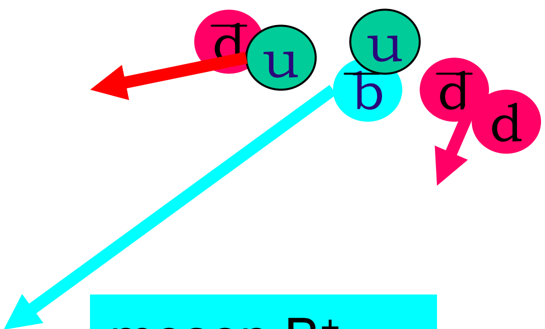
with the correct colours.

$\rightarrow$ at the end of the chain, one gets non-coloured hadrons

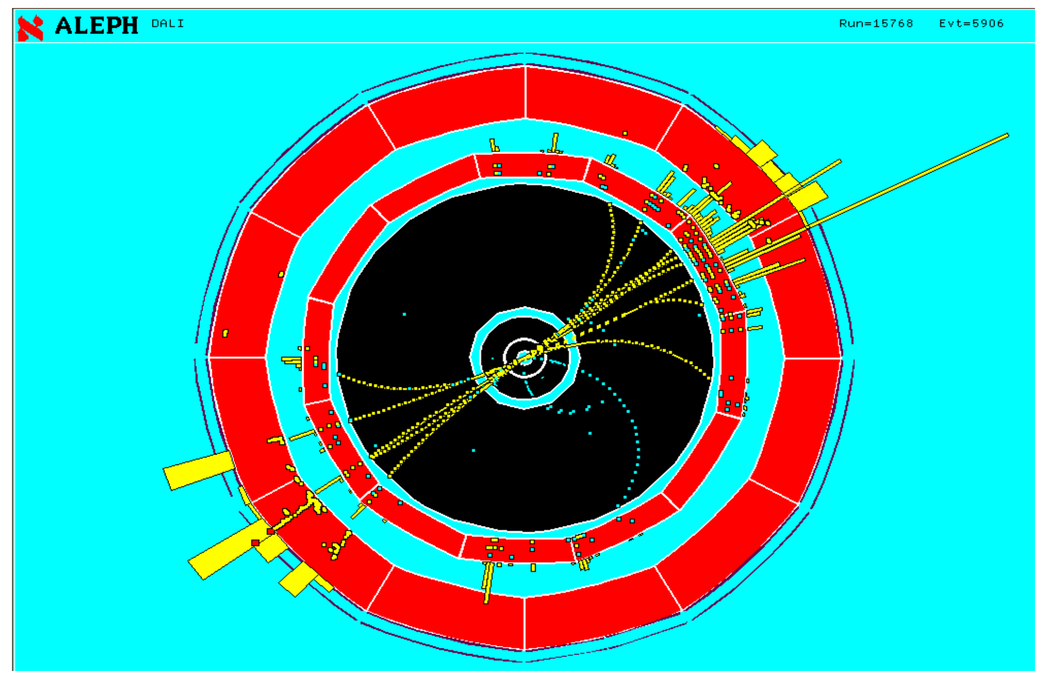
It is not only a model, one sees hadrons jets in the experiments !



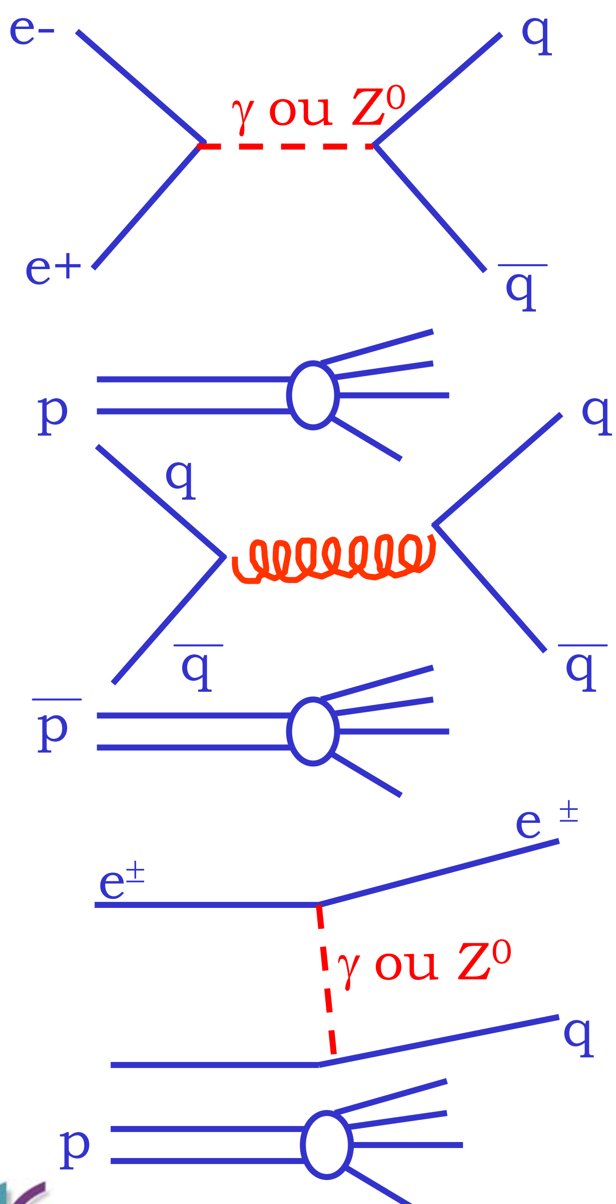
meson B_d

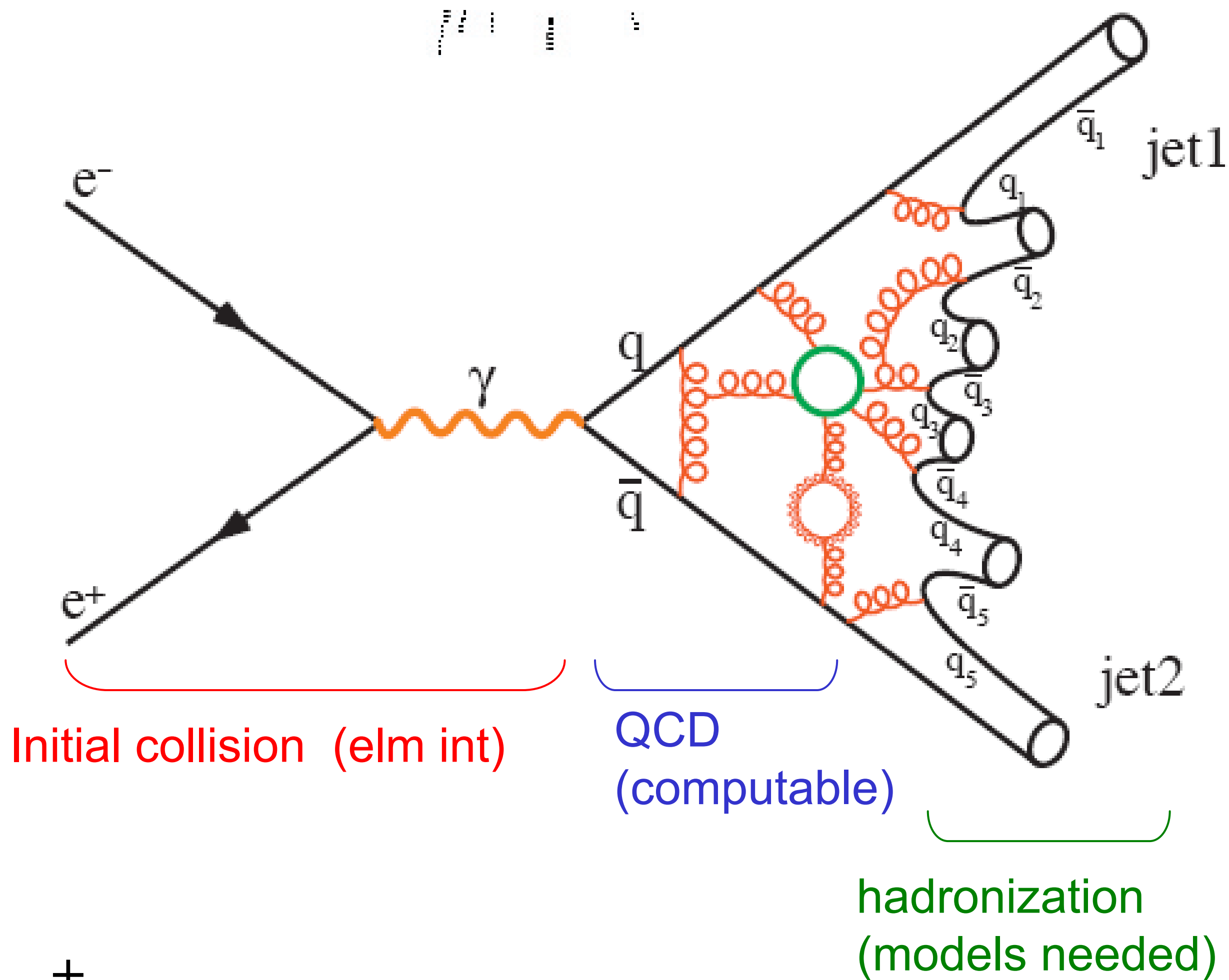


meson B^+



Examples of diagrams for quarks production in e^+e^-, pp or pe^- collisions





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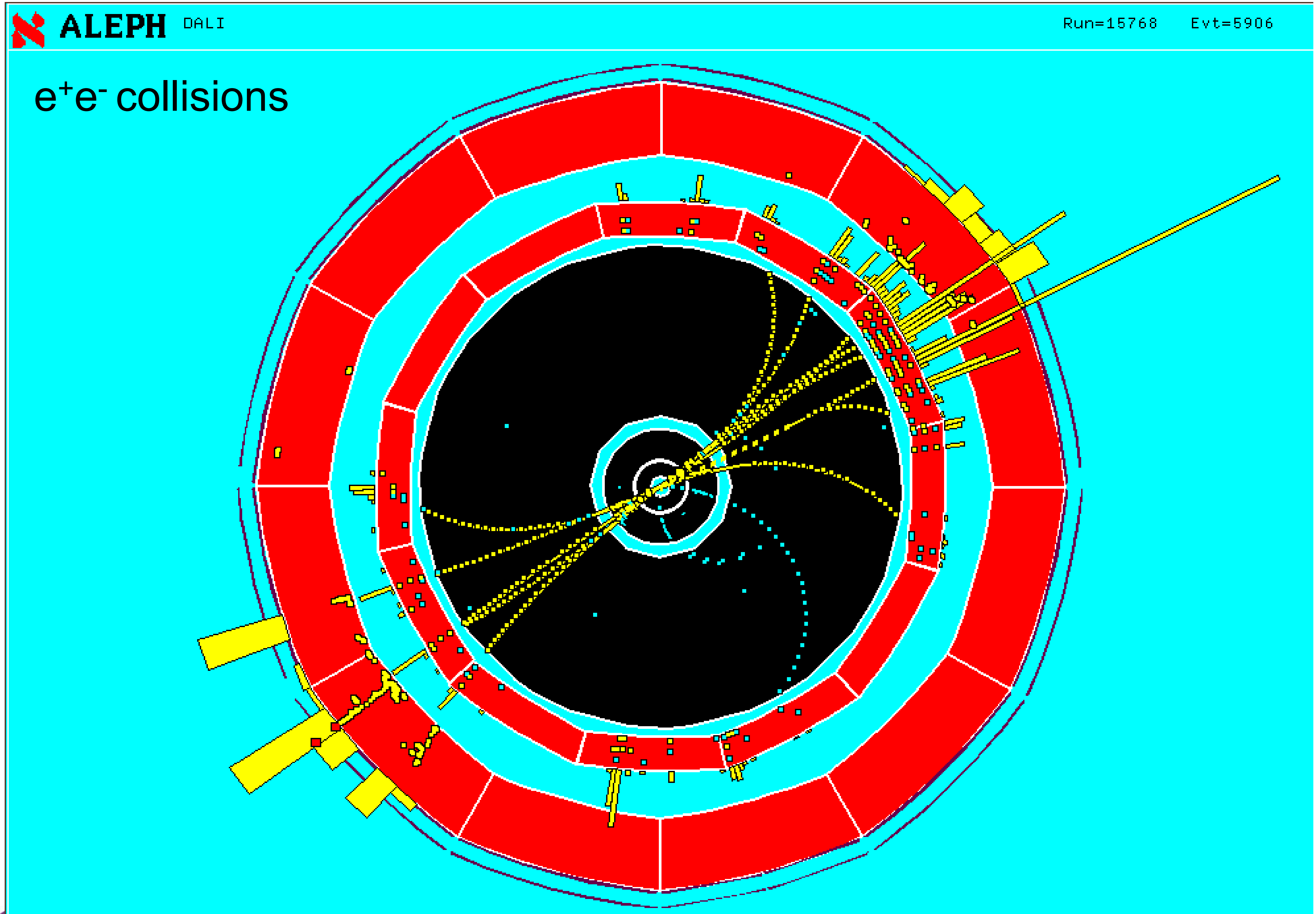
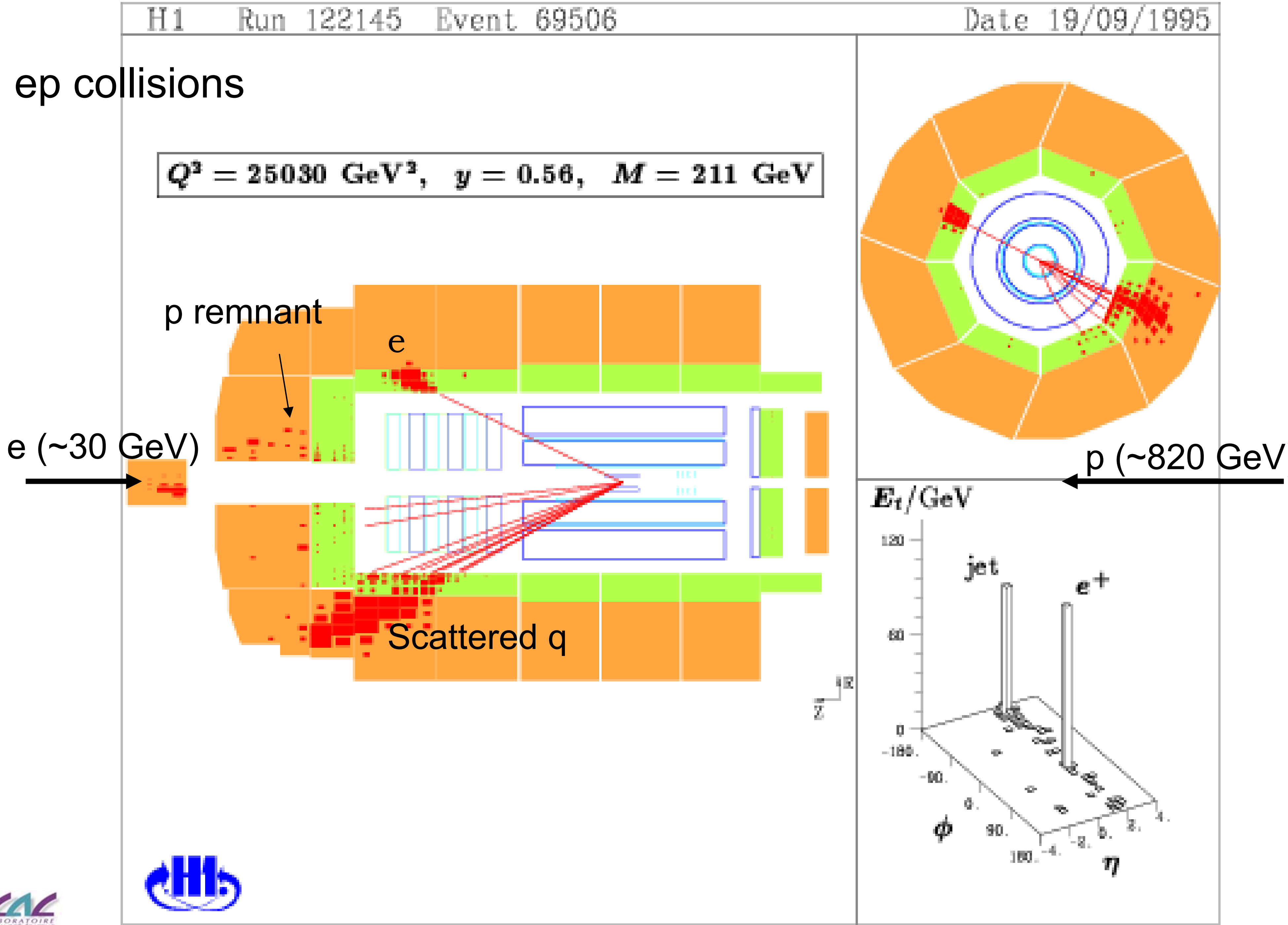
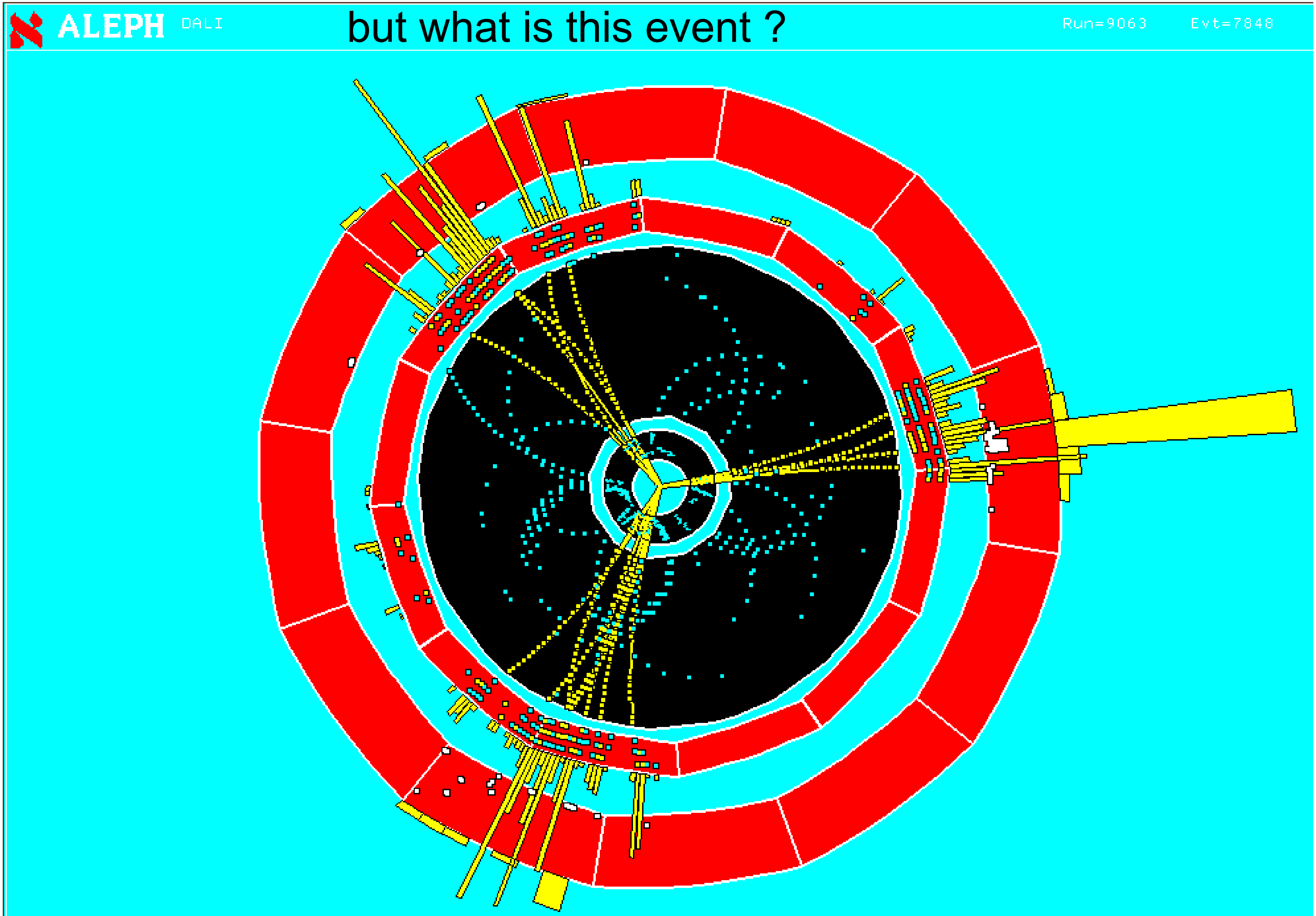

ALEPH
DALI
Run=15768
Evt=5906
e+e- collisions

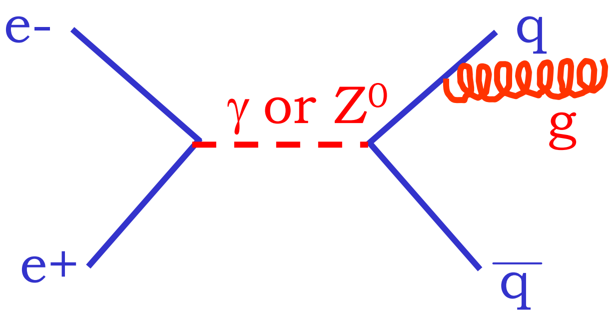
ep collisions

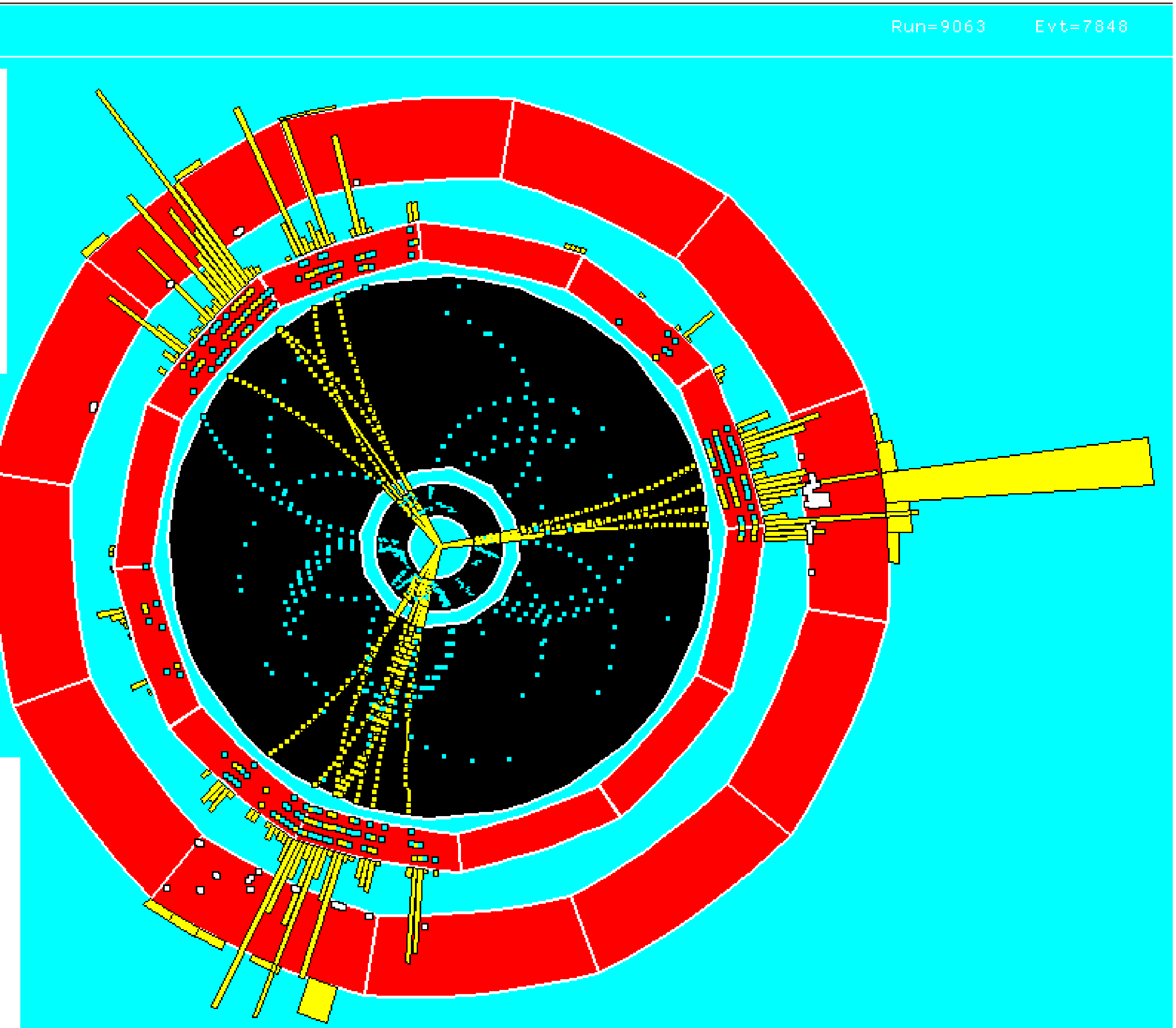


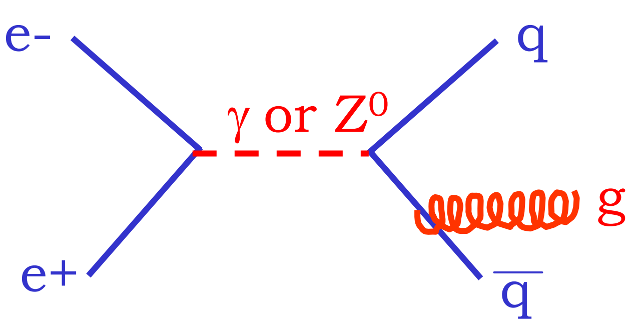
but what is this event ?



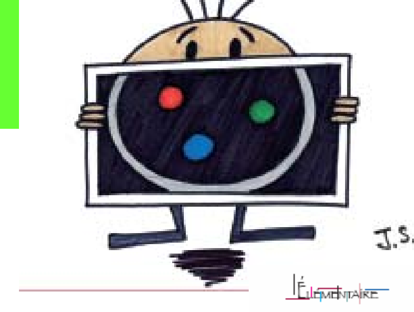
$e^+e^- \to q\bar{q}g$ 3 jets experimental evidence for the gluon







We have seen that …



- SU(2) [u$\leftrightarrow$d] symmetry is a very good symmetry
- SU(3) [u$\leftrightarrow$d $\leftrightarrow$s]] symmetry is a less good symmetry

- Experimentally one has shown that quarks are real and are the internal components of the hadrons :
  - measurement of the intrinsic magnetic moment of the proton
  - elastic and inelastic scatterings

- colour-SU(3) : exact local gauge symmetry (QCD) : 3 colours

  8 gluons carrying colours (self-interaction) they mediate the strong interaction

Only « white" particles can be seen (only qqq and $q\bar{q}$)

- The « running" of the coupling constant α_s differs from the running in QED

  $\alpha_s \searrow$ when E $\nearrow$: asymptotic freedom

  $\alpha_s \nearrow$ when E $\searrow$: quarks confinement $\Rightarrow$ low energy : non perturbative regime, cannot perform series expansion , models are needed

In fact we will meet the strong interaction again : it will have to be understood in order to extract from the measurements the effects of weak interaction (CP violation)

Back up slides

The states of I_3=0, Y=0 of $3\otimes\bar{3}$

- The states A,B et C are linear combinations of uu, dd and ss

- The SU(3) singlet should contain the combination which has the 3 same weights for uu, dd and ss (as we did for spin) :

$$\left|I=0, I_3=0\right\rangle = \sqrt{1/2}\left(u\bar{u}+d\bar{d}\right)$$

$$\eta_1 = C = \sqrt{\frac{1}{3}}\left(u\bar{u}+d\bar{d}+s\bar{s}\right)$$

- A is chosen to belong to the triplet (du,A,-ud) :

$$\pi^0 = A = \sqrt{\frac{1}{2}}\left(u\bar{u}-d\bar{d}\right)$$

- The eigenstates of an hermitian operator are orthogonal and its eigenvalues are real => the isospin singlet B has to be orthogonal wrt A and C and is :

$$\eta_8 = B = \sqrt{\frac{1}{6}}\left(u\bar{u}+d\bar{d}-2s\bar{s}\right)$$